The Journal of Psychology and Financial Markets
2001, Vol. 2, No. 2, 80–99

Financial Bubbles: Excess Cash, Momentum, and Incomplete Information

Gunduz Caginalp, David Porter, and Vernon Smith

We report on a large number of laboratory market experiments demonstrating that a market bubble can be reduced under the following conditions: 1) a low initial liquidity level, i.e., less total cash than value of total shares, 2) deferred dividends, and 3) a bid–ask book that is open to traders. Conversely, a large bubble arises when the opposite conditions exist.

The first part of the article is comprised of twenty-five experiments with varying levels of total cash endowment per share (liquidity level), payment or deferral of dividends and an open or closed bid–ask book. We find that the liquidity level has a very strong influence on the mean and maximum prices during an experiment (P < 1/10,000). These results suggest that within the framework of the classical bubble experiments (dividends distributed after each period and closed book), each dollar per share of additional cash results in a maximum price that is $1 per share higher.

There is also limited statistical support for the theory that deferred dividends (which also lower the cash per share during much of the experiment) and an open book lead to a reduced bubble. The three factors taken together show a striking difference in the median magnitude of the bubble ($7.30 versus $0.22 for the maximum deviation from fundamental value).

Another set of twelve experiments features a single dividend at the end of fifteen trading periods and establishes a 0.8 correlation between price and liquidity during the early periods of the experiments. As a result, calibration of prices and evolution toward equilibrium price as a function of liquidity are possible.

Introduction

Financial markets often exhibit sharply rising prices and subsequent declines that cannot be justified by fundamental or realistic economic assessments (Dreman and Lufkin, 2000). But the recent dramatic rise and fall of Internet-related technology shares have demonstrated that such spectacles are not regulated to distant eras. The immediate availability of information about every publicly traded company, along with omnipresent media analysis, seems to have done nothing to diminish the magnitude of bubbles.

The spectacular valuations of late 1999 and early 2000 have been well documented, and appear to be greater than those of the South Seas bubble in the 1600s (Dreman, 1998, Shiller, 2000). Despite the fact that the availability and diffusion of information has improved incomparably, this most recent bubble (for a large number of stocks) attained price levels that were over 100 times their realistic valuation, even under the most optimistic estimates. This underscores the fundamental behavioral nature of the bubble phenomenon, and casts doubt on the thesis that major bubbles are the result of poor availability of information. The enigma of bubbles has inspired many laboratory experiments demonstrating the robustness and the endogenous aspect of boom–bust cycles.

Laboratory asset market experiments in economics are an increasingly important tool in understanding markets. These experiments usually comprise a number of participants, who are given a combination of one or more assets whose payouts are prescribed by the experimenters. While in early experiments, as in early exchanges, the participants arranged deals on their own or posted them on a blackboard, current experimental asset markets are usually executed through a computer network, using any one of numerous auction mechanisms (see, for example, Van Boening, Williams and LaMaster, 1993 for a discussion of auction methods, and Davis and Holt, 1993 or Smith, 1982 for experimental economics in general).

The laboratory markets are an important complement to studying market phenomena through field

Gunduz Caginalp is a professor in the Mathematics Department at University of Pittsburgh.

David Porter is a professor in the College of Arts and Sciences at George Mason University.

Vernon Smith is a professor in the Department of Economics and Law at George Mason University.

Requests for reprints should be sent to: Gunduz Caginalp, Mathematics Department, University of Pittsburgh, Pittsburgh, PA 15260. Email: caginalp@pitt.edu

data, because hypotheses can be tested by defining appropriate rules of payout for the asset and then replicated. In particular, the feasibility of trading across periods, during which the fundamental value of the asset may change, leads to the possibility of studying price dynamics in markets.

One experiment offers a particularly clear and simple challenge to the basic efficient market hypothesis, and thus has been replicated many times. It involves a single asset that pays a dividend with a fixed expectation value each period (see, for example, Smith, Suchanek and Williams, 1988 and Lei, Noussair and Plott, 1998). The participants are told that the asset will pay a dividend with an expected value of 24 cents at the end of each of the fifteen periods, and will subsequently be worthless. Hence, the fundamental value of the asset is \$3.60 during the first period and declines by 24 cents in each successive period until the end of the fifteenth period, when it is worthless. Traders are given an endowment consisting of some shares of the asset and some cash. Throughout the trading periods, they can trade by placing or accepting orders on the computer network.

Classical economics predicts that the trading prices will fluctuate in a tight range near a fundamental value that is commonly known. In fact, in most of these experiments, the expected value of the asset is displayed on the trading screen. Many sets of experiments under a variety of conditions have shown that prices often start lower than the \$3.60 fundamental value during the first period, and rise far above the fundamental value during the middle to late periods. Sometime between the eleventh and fifteenth periods the asset price begins to crash and usually goes below fundamental value. A variety of auction mechanisms have been used to match up the bids and offers, with the same result.

These replicable experiments thus differ sharply from any prediction that could be made from the available theories. Possible explanations center on the features of world markets that were not represented in the experiments, such as short selling, margin buying and transaction costs. But further experiments showed that none of these features eliminated or significantly reduced the price bubble (see Porter and Smith, 1994 for a review). Experiments under different conditions, such as equality of endowments and complete certainty of dividend draws, and even a subject pool consisting of businesspeople in place of undergraduates, also did not diminish the bubble.

But the bubble was diminished significantly by one factor: experience in trading with the same group (Smith, Suchanek and Williams, 1988). When the same traders were brought back for a second experiment, the magnitude of the bubble diminished significantly. During a third experiment, the bubble was eliminated entirely and prices remained close to fundamental value.

As noted by Smith, Suchanek and Williams [1988], the traders know all the information about the asset, so the only source of uncertainty involves the future actions of the other traders. The strategies of other traders are manifested in the price change each period after the first. As prices rise beyond the fundamental value, the traders become aware that other traders are making decisions based on factors beyond valuation alone. This feature cannot be explained by classical price theory, because it assumes that each trader will not only self-optimize but will *rely* on the self-optimization of others.

This basic idea was discussed within the context of specific experiments by Beard and Beil [1994], who showed that the reliance on the self-optimization of others is not always a valid idealization. In the context of the bubble experiments, the deviation of the price from the fundamental value reveals explicit information that other traders are not engaging in idealized game theoretic behavior based upon fundamental value. Rather, at least some of the traders are using a momentum strategy, e.g., placing orders with the expectation of a continued rise in prices. Consequently, even the traders who had not planned to implement a momentum strategy are forced to recognize it as an important factor in determining the temporal evolution of prices.

The neoclassical theories of price dynamics assume that price changes occur only in response to a deviation from the fundamental value of the asset (see, for example, Watson and Getz [1981]). Momentum trading is incorporated in a particular model only if the demand and supply are dependent in part on the price change, or derivative, of the asset price. This theory has been discussed in several papers (Caginalp and Balenovich, 1999; Caginalp, Porter and Smith, 2000a and references therein) using a differential equations model that incorporates supply/demand considerations for value-based and trend-based (or momentum) sentiment. From the perspective of this differential equations model, an initially undervalued price spurs buying from the value-based sentiment. This creates an uptrend that eventually induces momentum, creating a sentiment to buy even after prices have exceeded the fundamental value and despite some selling by the value-based investors. This uptrend continues until the momentum traders have an inadequate amount of cash, at which point prices plateau and begin to decline. Once the decline begins, momentum sentiment to sell is spurred, and prices often fall precipitously.

The implications of this differential equations model have been examined statistically, and the "out-of-sample" forecasting capabilities for laboratory experiments have been compared with other possible theories (Caginalp, Porter and Smith, 2000b). For example, one implication is that a low initial price tends to result in a larger bubble, because the initial undervaluation spurs strong buying due to fundamental reasoning. This rapid rise in prices causes an enhanced momentum effect that leads to a bigger bubble.

This prediction has been confirmed experimentally by using "price collars," or constraints on price movements during the initial trading period (Caginalp, Porter and Smith, 2000a), where the differential equations model has also been adapted to provide forecasts of the trading prices one and two periods ahead. These predictions were compared with 1) time series predictions, including random walk and pure momentum, 2) the excess-bids model considered in Smith, Suchanek and Williams [1988], and 3) human forecasters. In general, the differential equations provide the best analytical forecasts for two periods ahead, and are comparable to the best human forecasters who had participated in these experiments previously. The time series method using ARIMA (autoregressive integrated moving average), with a coefficient halfway between pure random walk and pure momentum, is the most efficient analytical forecasting method for one period ahead.

The differential equations model focuses on the equation for price change per unit of time, which is determined by the imbalance in supply and demand of the asset. Within our approach, the fundamental value and price momentum influence price through the net ratio of supply and demand. In particular, if there is a large supply of available cash compared to the shares of the available asset, there should be a greater tendency for prices to rise versus the opposite situation. This is a key factor in markets that draws the attention of practitioners.

For example, in underwriting an initial public offering (IPO) or a secondary public offering, there is the important issue of the "float" and whether the supply of cash likely to be committed to the issue will be large or small compared to the supply of stock to be sold. While investment houses have long known that an excess supply will lead to artificially low prices, there has been no way to account for this within classical economic theory.

This concept became increasingly important as the general public flocked to IPOs related to Internet technology companies during 1999 and 2000. In some cases, insiders already owned a large percentage of the shares, so only a relatively small fraction were sold to the public. But at the same time there was a huge public appetite for these shares, as instant riches from one IPO led to a greater frenzy for the next. This severe imbalance between the available cash and the available supply led to prices that sometimes increased up to 1,000% on the first day of trading (e.g., VA Linux in late 1999). Excess cash, or liquidity as it is sometimes called, is an important factor in many bubbles because it provides the fuel for excessive price rises. While a steep uptrend in prices increases positive sentiment among momentum traders, the extent of further price increases is determined in part by the available cash within this group relative to the size of the supply.

There is considerable reason to believe that the relative amount of excess cash or liquidity has a strong bearing on price evolution, but this effect, like momentum, is absent in classical price theory. In an effort to quantify this effect in the laboratory, Caginalp, Porter and Smith [1998] performed a series of seven asset market experiments. Nine participants were given the opportunity to trade an asset whose sole value consisted of a dividend with an expectation value of $3.60 at the end of the fifteen-period experiment. Each participant was given a distribution of cash and asset at the beginning of the experiment. The auction mechanism consisted of a sealed bid-offer (SBO). This double auction mechanism allows buyers to submit bids and sellers to submit offers (Davis and Holt, 1993). The bids are arrayed from high to low as a demand function, and the offers are likewise arrayed from low to high as a supply function. The intersection of the supply and demand is determined as the price. If the bid and ask arrays overlap vertically, the price is determined to be the average price in the region of overlap. All offers below this trading price are sold at the intersection price, while those above it are rejected. Similarly, all bids above the price are executed at the intersection price, while those below it are rejected.

At the start of the experiment, the traders were told that there would be a single payout at the end of the fifteenth period, with a 50% probability of a $3.60 payout, and a 25% probability each of either a $4.60 or a $2.60 payout. The seven experiments differed only in the total amount of cash relative to the total amount of assets.

In three of the experiments, the participants received more total cash, denoted D, than the total number of the asset multiplied by the expectation value of $3.60, denoted S. In the other four experiments, there was a slight excess supply of asset. In particular, the ratio $q = (S - D)/S$ was –0.86 for the cash-rich experiments and 0.125 for the asset-rich experiments. In the three cash-rich experiments, the first period prices were $5.91, $5.05 and $7.64. Hence, in each cash-rich experiment, the first period price exceeded even the highest possible payout for the asset (namely $4.60).

The four asset-rich experiments exhibited first period prices of $4.99, $4.03, $2.88 and $2.89, so that the highest of the asset-rich prices remained below the lowest of the cash-rich prices. Statistical testing of these values and those of the mean and median prices during the entire experiment led to the strong conclusion that prices in cash-rich experiments were higher than those in asset-rich experiments.

It is also interesting to note that the trading price for each period gradually approached fundamental value (which is constant at $3.60 for the entire experiment) toward the end of the experiment. This provides some consolation to the rational expectations theory. However, since all the information is known at the beginning of the experiment, the length of time necessary to attain fundamental value is incompatible with classical theory. Furthermore, what is the nature of this return to equilibrium, and what is the role of excess cash, or liquidity, in this process, and the associated time scale for this process?

We discuss two sets of experiments to address these questions. The first set, called declining fundamental value, tests the effect of excess cash using the typical bubble experiment conditions. That is, participants trade an asset that pays a dividend with an expectation value of 24 cents each period for fifteen periods. In these experiments, we examine the extent to which the excess cash results in a bubble of larger magnitude. We also consider the effect of deferring the dividends until the end of the experiment to see if the absence of additional cash during the experiment leads to a dampening of the price bubble. In a subsequent paper, we study this additional liquidity issue explicitly with the differential equations approach.

Another issue tested within these experiments is whether an "open book," in which traders can see the array of orders (but not the identity of the traders), leads to lower prices than "closed book" trading.

In the second set of experiments, which we call single payout, the asset pays a single dividend at the end of the experiment. This minimizes the effects of momentum, and the effect of liquidity can be calibrated by varying the initial cash/asset ratio. These experiments also confront some of the problems inherent in IPOs and closed-end funds.

The paper is organized as follows. The next section describes the first set of experiments, and we analyze them in the subsequent section. We then report on the single payout experiments and perform statistical analysis. Our aim is to determine the average increase in the trading price of the asset for each additional dollar of excess cash per share that is endowed at the beginning of the experiment. The results and implications for world markets are discussed in the Conclusion.

Bubble Experiments (Declining Fundamental Value) With Varying Conditions

We report on a set of twenty-five experiments conducted at the University of Arizona between March and December 2000. In each experiment, between nine and twelve participants were recruited from undergraduate students who had not previously participated in a related asset market experiment. The computerized instructions (see the Appendix) familiarized the participants with the trading mechanism and informed them of the rules for the single asset to be traded through the computer network. The instructions describe the auction procedure, along with a graphical illustration of the matching of orders to obtain the trading price.

The asset paid a dividend with an expectation value of 24 cents during each period (with draws of 0, 8, 28 or 60 cents, each with a 25% probability). Each trader was given an allotment of asset and cash. The total amounts of cash and asset varied with each experiment.

In all of the experiments, there were fifteen trading periods lasting two minutes each, during which each trader could place orders to buy and/or sell the asset. The orders could be changed or withdrawn prior to the end of the trading period. At the end of each period, the program matched the orders in accordance with the sealed bid-offer (SBO) double auction (described in Van Boening, Williams and LaMaster, 1993). Each experiment also designated either a closed book (CB) or an open book (OB) procedure, to test whether this information, if available to the traders, tends to diminish the size of the bubble:

- Closed Book (CB). In the standard bubble experiments of this type, the traders do not see the other orders as they enter their own orders; they only see the resulting price and the volume.
- Open Book (OB). All orders (but not the identity of the trader placing the trade) are visible on the screen to all participants.

Smith, Suchanek and Williams [1998] have noted that near the peak of the price bubble there is a sharp drop in the number of bids. Thus, prices are rising, with fewer traders buying shortly before the crash. This acts as a precursor to the bursting of the bubble, and indicates that information from the trading history could be useful in forecasting the peak.

At the end of each period of trading, the participants are also notified of the dividend draw. The computer program allows the experimenter to choose between two options regarding dividends:

- Dividends Paid (DP). This is the standard payout at the end of the period, and allows the cash to be used for trading throughout the remainder of the experiment.
- Dividends Deferred (DD). The trader who holds the shares at the end of the period is entitled to the dividend, but does not receive the cash until the end of the entire experiment. Hence the cash cannot be used for trading during the remainder of the experiment.

In the DP case, our basic hypothesis stipulates that we expect the additional cash to raise the average trading price to some extent throughout the periods.

In each of the experiments, the most important designation is the total initial cash allotment to all traders in comparison with the total asset allotment as designated by one of these three options:

• Even Cash/Asset Ratio (ER). The total amount of cash distributed is equal to the value of the total amount of assets distributed. If there are N traders, there is a total allotment of $10.80 × N in cash, and 3N shares with a fundamental value of $10.80 × N. The individual allotment of cash is $7.20 for the first three traders, $10.80 for the next three traders and $14.40 for the next three traders. If there are more than nine traders (with a maximum of twelve), the remaining traders receive a cash allotment of $10.80. The asset amounts are 4, 3 and 2, respectively, for the three groups, with any remaining traders allotted 3 shares each.

• Cash-Rich Ratio (CR). The total amount of cash distributed is twice the value of the total amount of assets distributed to all participants. If there are N traders in the experiment, the total amount of cash is $14.40 × N, while the number of assets is 2N with a valuation of $7.20 × N. The individual allotments are similar to ER. In this cash-rich case, the analogous amounts are $10.80, $14.40 and $18.00 in cash, plus 3, 2 and 1 share(s) each, respectively, for the three groups of traders. Hence the initial cash distribution is twice the value of the initial asset valuation.

• Asset-Rich Ratio (AR). The total amount of cash distributed is half the value of the total amount of assets distributed to all participants. The total amount of cash is $7.20 × N, while the number of assets is 4 × N with a valuation of 14.40 × N. The individual allotments are again similar to ER. In this asset-rich case, the analogous amounts are $3.60, $7.20 and $10.80 in cash, plus 5, 4 and 3 shares each, respectively, for the three groups of traders. Hence the initial cash distribution is half the value of the initial asset valuation.

The experiments using the single payout dividend (Caginalp, Porter and Smith, 1998) suggest that the magnitude of a bubble can be affected by varying the initial cash/asset ratio, i.e., the AR designation would lead to a bubble of larger magnitude than the CR.

In summary, we have three variables that can be adjusted for each experiment, CB/OB, DP/DD and ER/CR/AR, leading to twenty-four distinct combinations. Our hypothesis is that the largest bubble would arise under conditions CR/DP/CB, i.e., an initial cash-rich endowment with dividends paid each period (adding to the excess cash), and a closed book trader screen. We expect the smallest bubble if conditions AR/DD/OB are implemented. Among the twenty-five experiments, we compare three in the CR/DP/CB and three in the AR/DD/OB cases below.

Table 1 displays the trading prices for each period for all twenty-five of the experiments, together with the designation in terms of the variables defined above, and the mean maximum trading price. For each experiment, we subtract from the trading price $P(t)$ for each period the fundamental value $P_a(t)$. The latter is simply $3.60 minus 24 cents times the period number. For each experiment we list the maximum of the differences $P(t) - P_a(t)$, denoted MaxDevPrice, as another indication of the size of the bubble.

In comparing the three CR/DP/CB experiments with the three AR/DD/OB experiments we find that the maxi-

Table 1a. *All 25 Declining Value Experiments With Summary Statistics—Part 1*

Period	Fund Value	110300 1	1101obar	110300 2	A031000	1031ar2	1031ar	A030900	92700obb l2ar
1	360	80	100	60	60	50	50	80	80
2	336	70	105	100	70	60	60	100	95
3	312	90	112	107	100	75	70	115	100
4	288	85	60	127	105	83	85	125	99
5	264	86	50	245	120	90	100	145	110
6	240	91	65	270	145	95	140	200	145
7	216	100	95	275	160	110	200	255	125
8	192	91	100	196	180	125	300	300	137
9	168	92	95	200	192	141	330	300	280
10	144	98	86	174	195	159	340	300	155
11	120	100	95	101	170	181	340	285	180
12	96	91	98	119	135	200	299	240	245
13	72	88	80	100	65	211	150	185	150
14	48	54	70	54	45	199	20	75	113
15	24	35	45	23	18	160	10	60	73
Mean	180	83	84	143	117	129	166	184	139
Maximum	360	100	100	275	195	211	340	300	280
Max Deviation	Not Appl	16	22	59	51	151	220	165	149
Liquidity	Not Appl	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
Dividend Distributed	Not Appl	0	0	0	0	0	0	0	1
ClosedBk	Not Appl	0	0	0	1	1	1	1	0

Table 1b. *All 25 Declining Value Experiments With Summary Statistics—Part 2*

Period	Fund Value	92700oba l3ar	ar5300	92600 ob l4ar	92900ob l0 ar	92900 ll ar	A030800	A030600	A030100
1	360	68	100	100	63	65	55	60	70
2	336	79	80	110	60	80	67	80	82
3	312	92	83	121	65	95	100	110	120
4	288	155	100	121	90	113	250	146	130
5	264	190	120	200	111	135	280	210	150
6	240	260	160	281	125	162	275	250	170
7	216	349	201	260	142	198	230	248	190
8	192	210	188	300	150	200	200	239	225
9	168	210	210	300	141	190	160	230	220
10	144	215	214	290	125	190	146	220	230
11	120	201	440	270	155	170	110	210	240
12	96	160	400	290	136	145	110	195	235
13	72	190	444	260	144	125	75	150	220
14	48	148	150	140	115	108	25	20	180
15	24	80	100	100	95	78	20	26	26
Mean	180	174	199	210	114	137	140	160	166
Maximum	360	349	444	300	155	200	280	250	240
Max Deviation	Not Appl	133	372	194	72	60	35	99	148
Liquidity	Not Appl	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
Dividend Distributed	Not Appl	1	1	1	1	1	1	1	1
ClosedBk	Not Appl	0	0	0	1	1	1	1	1

Table 1c. *All 25 Declining Value Experiments With Summary Statistics—Part 3*

Period	Fund Value	E1207_1	E1207_2	042800c	042600c	92900ob l0 cr	92100ob 11cr	Cma0127	C1208_1	Cfe0121
1	360	110	100	100	120	180	105	100	100	90
2	336	130	130	130	133	208	240	237	100	160
3	312	170	150	150	170	302	335	250	125	250
4	288	230	200	201	261	496	610	300	155	300
5	264	264	250	352	437	500	562	320	200	400
6	240	260	300	240	655	415	445	330	255	500
7	216	216	300	302	410	476	548	350	300	600
8	192	180	250	309	650	435	420	400	375	700
9	168	180	225	302	650	446	350	440	460	775
10	144	150	200	375	400	411	261	500	550	825
11	120	140	200	315	482	497	330	560	630	850
12	96	110	150	314	350	498	300	600	720	800
13	72	100	150	352	120	330	290	300	800	200
14	48	85	125	275	156	125	300	200	830	180
15	24	52	75	175	140	21	390	75	0	100
Mean	180	158	187	259	342	356	366	331	373	449
Maximum	360	264	300	375	655	500	610	600	830	850
Max Deviation	Not Appl	28	84	280	482	402	366	504	782	730
Liquidity	Not Appl	3.6	3.6	7.2	7.2	7.2	7.2	7.2	7.2	7.2
Dividend Distributed	Not Appl	1	1	1	1	1	1	1	1	1
ClosedBk	Not Appl	1	1	0	0	0	0	1	1	1

Note: The trading price (single bid-offer) for each period is displayed for each of 25 (declining fundamental value) experiments. Displayed below are the mean price, the maximum price and the maximum deviation from fundamental value for each experiment. For each experiment the three parameters are shown: Liquidity (total cash divided by the total number of the asset), Dividends Distributed (equals 1 if the dividends are distributed each period and O if they are deferred) and Closed Book (equals 1 if traders do not see others' orders, and 0 if they see all orders placed). The data show low prices and no bubbles when $L = 1.8$ (half as much cash as asset), the dividends are deferred with an open book. When $L = 7.20$, dividends are paid at the end of each period and traders do not see all orders, there is a large bubble as prices rise five or more dollars above fundamental value.

mum deviations from fundamental value are 782, 730 and 504, respectively, with an average of 672, much larger than the 22, 16 and 59, respectively, for the latter set of experiments, which have an average of just 32. Hence there is a factor of almost 21 between the two sets of conditions. Figure 1, which displays these prices for the experiments at the two extremes defined above, also suggests that the bubble is much more pronounced when the set of former conditions apply. We examine next the statistical questions of whether each of these variables influences the magnitude of the bubble.

Statistical Analysis (Mixed Effects and Regression)

We perform a multivariable linear regression in terms of the predefined sets of independent variables. Let *L* (or liquidity) denote the total cash allotment divided by the total asset value at the start of the experiment, so that *L* = \$3.60 for the even cash case (ER), *L* = \$7.20 for the cash-rich case (CR) and *L* = \$1.80 for the asset-rich case (AR). Caginalp and Balenovich [1999] note that this liquidity price (with units of dollars per share) is another important price per share beyond the trading price and the fundamental value per share. We use the numerical designations 1 for the dividends paid case (DP) and 0 for the dividends deferred case (DD). Similarly, we let 1 denote the closed book case (CB), and 0 the open book case (OB).

We perform a regression of the mean price for each experiment with respect to these three variables using Minitab 11.2 software. The result is the regression equation:

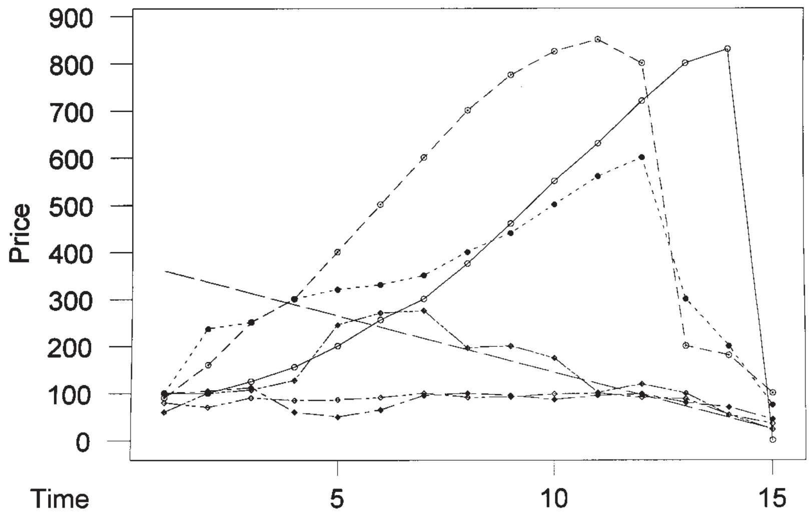
$$\textit{MeanPrice} = 59.8 + 36.5\,\textit{Liquidity} + 23.1\,\textit{DivDistr} + 7.1\,\textit{ClosedBk}$$

Each coefficient has the positive sign indicated by our hypotheses. The coefficient of *L* is 36.5 with a standard deviation of 4.1, resulting in a *T*-value of 8.87 and a P-value of less than 1/10,000. This provides very strong statistical confirmation that excess cash results in significantly higher prices. The regression equation suggests that for each dollar of additional cash per share (i.e., for each additional \$1 rise in *L*) we see a 36.5 cent increase in the average price throughout the experiment. The amount of increase in price per additional dollar of excess cash is explored further in the next section, in the context of another set of experiments, that feature constant fundamental value.

The coefficient of 23.1 for the dividends distributed variable has a standard deviation of 21.7, resulting in a *T*-value of 1.06 and a *P*-value of 0.3. This provides some statistical evidence that distributing rather than deferring dividends tends to elevate prices. The coefficient of 7.1 for the closed book variable is 4/10 of a standard deviation away from the null hypothesis of zero, providing weak evidence ($P = 0.69$) that an open book diminishes a bubble. The constant coefficient has a *T*-value of 2.8 with $P = 0.01$. The analysis of variance results in an *F*-value of 36.4, with *P* less than 1/10,000.

To further substantiate these results, we implement the linear mixed effects model (S-Plus 2000 software).

FIGURE 1
Price Evolution Under Conditions Maximizing and Minimizing Bubbles



Note: The price evolution is shown for six experiments, along with the straight line representing the fundamental value (which declines from \$3.60 to \$0.24). In the three experiments, marked by circles, in which prices soar far above the fundamental value, there is an excess of cash, the dividends are distributed at the end of each period (adding more cash) and there is a closed book so that traders do not know the entire bid–ask book. In the experiments marked by diamonds, the opposite conditions prevail, and prices remain low and there is no bubble.

With the trading price as the dependent variable, and liquidity, deferred dividends and closed book as the independent variables, we obtain similar results. In particular, the coefficient of liquidity is 34.57, with a standard error of 3.86, a T-value of 8.95 and $P < 0.001$. The deferred dividends variable has a coefficient of 22.53 and a standard error of 20.35, with a T-value of 1.11 and $P = 0.28$. The closed book variable has a value of 1.38 and standard error of 16.69, with a T-value of 0.083 and $P = 0.93$. Hence the mixed effects model provides a slightly stronger confirmation of the effect of liquidity on price than the previous confirmation for the role of deferred dividends.

Next we examine the statistical difference among particular groups of experiments: the CR/DP/CB favoring higher prices and larger bubbles, versus AR/DD/OB favoring lower prices and smaller bubbles (see Figures 1 and 2).

The mean of the average trading price of each experiment in the CR/DP/CB group is 384.3 with a standard deviation of 59.7, while the mean of the AR/DD/OB group is 103.5 with a standard deviation of 34.5. The difference between the two groups is very significant, as shown by the statistical tests presented in the Appendix.

In summary, we have a compelling statistical validation of the hypothesis that these factors, taken together, can be used to magnify or reduce the size of a bubble very significantly. In each of the statistical tests above, there is only one data point used per experiment, thereby avoiding any possible problems with heteroscedasticity. In other words, the participants are the same throughout the experiment so that the most rigorous statistical criterion that can be implemented is the treatment of each experiment as a single observation.

The most important quantity from our perspective is the maximum deviation from fundamental value. Under the conditions we have identified as stimulating a large bubble (a high level of cash augmented by dividends paid each period and a closed book), the median maximum deviation of the trading price from fundamental value is \$7.30. For the opposite conditions, the trading price does not deviate by more than 22 cents from the fundamental value. In other words, the bubble is essentially eliminated by implementing all three conditions.

There is a weak statistical confirmation of the role of an open book in the size of the bubble for this set of experiments. It is possible that inexperienced traders have difficulty using the additional information in the order book. Further experimentation involving traders with some experience using the software could be useful to determine whether the open book has more of an impact on the magnitude of bubbles.

Next we consider subsets of the data, beginning with the closed book and dividends paid case, which are characteristic of a classical bubble experiment. The statistics presented in the Appendix indicate that within the framework of the classical bubble experiments (dividends distributed after each period and a closed book) each dollar per share of additional cash results in

1. A maximum price that is about \$1 per share higher;
2. An average trading price for the experiment that is about 45 cents higher;
3. A maximum deviation from fundamental value that is \$1.11 higher.

Thus, the magnitude of the bubble is strongly linked to the amount of additional cash.

In the open book case (with dividends distributed each period as before), each additional dollar per share of cash results in

1. A maximum price that is about 36 cents higher;
2. An average trading price that is about 28 cents higher;
3. A maximum deviation from fundamental value that is about 32 cents higher.

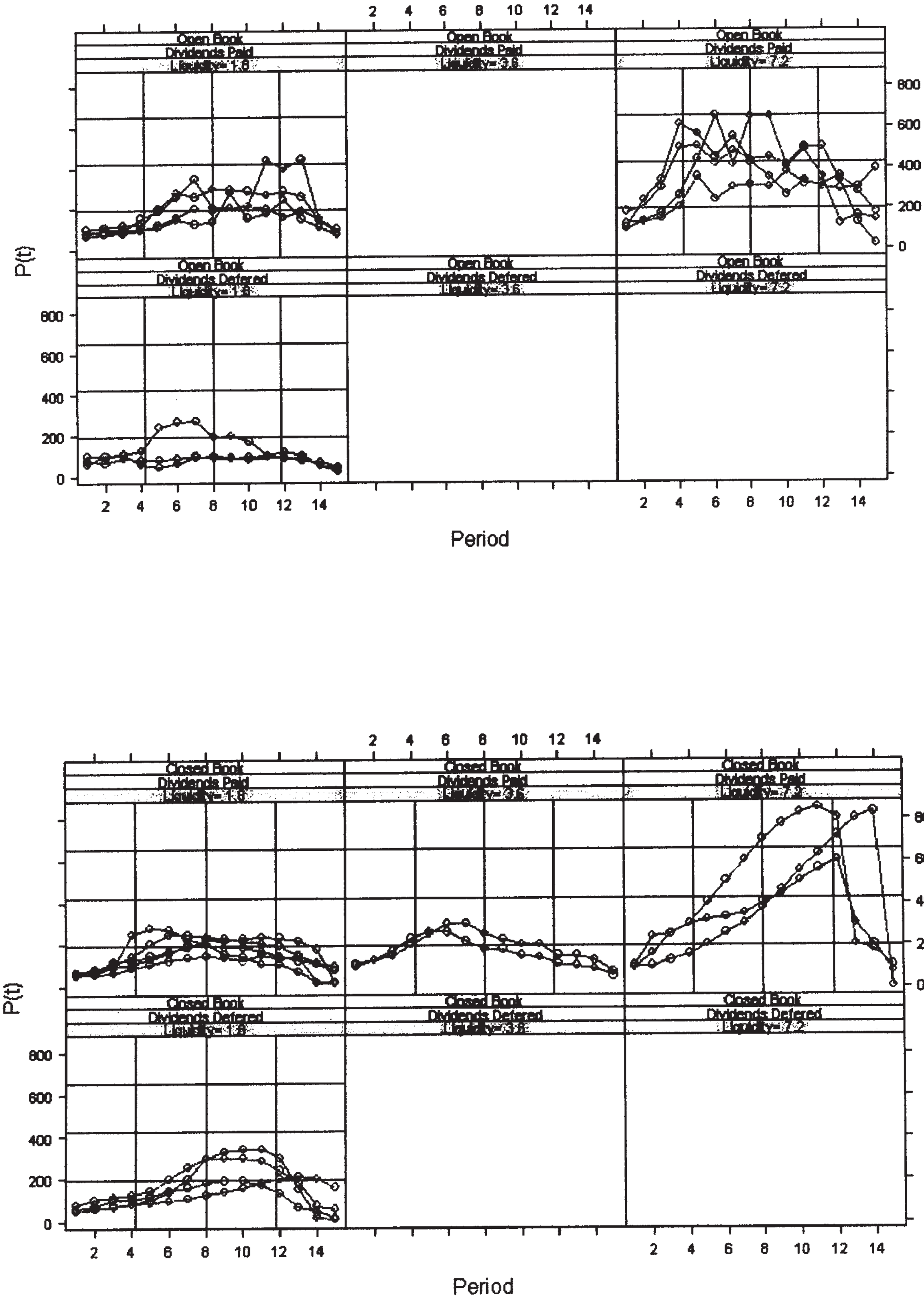
The maximum price and the maximum deviation from fundamental value are considerably lower than the corresponding values for the closed book case. Thus, the data suggest that the impact of additional cash is larger under closed book conditions.

Experiments With Constant Fundamental Value

The previous set of experiments shows an average increase in trading prices for each dollar per share of additional cash. In these experiments, however, there are other factors arising from the declining fundamental value of the asset. One way to focus more directly on the effect of additional cash in the system is to use a single payout experiment. This eliminates the role of exogenous changes in value and reduces the role of momentum.

The second set of twelve experiments again uses a sealed bid-offer (SBO), one-price clearing mechanism in each trading period and has the same framework as those in the previous section. The only difference is that the asset has a single dividend payout at the end of the fifteenth period. The dividend has an expectation value of \$3.60 (a 25% probability each of a \$4.60 and a \$2.60 payout, and a 50% probability of a \$3.60 payout). Traders were informed of the expected dividend at the start of the experiment. Each participant received an allotment of cash and shares and was able to trade with other participants in each of fifteen four-minute periods through a local area network. There were nine to twelve participants in each experiment. The subjects were undergraduates at the University of Arizona who

FIGURE 2
Price Evolution for Each of the Declining Fundamental Value Experiments



Note: The price evolution of each of the 25 declining value experiments is grouped in accordance with the three designations: liquidity value, dividends paid or deferred, and open or closed book.

had not participated in a related asset market experiment. The experiments were conducted during 1997 at the Economics Sciences Laboratory at the University of Arizona.

The experimental treatment among the twelve experiments differs only in terms of cash per share, or liquidity, L, which is defined as the (total) initial cash distributed to all participants divided by the total number of shares distributed (see Table 2). Thus, an experiment for which $L = \$7.20$ begins with twice as much cash as stock value (measured in terms of fundamental value, or \$3.60 per share). The price evolution is displayed for two typical experiments in Figure 3. We sort the experiments as cash-rich ($L > \$3.60$) or asset-rich ($L < \$3.60$), and compute the average of the fifteen prices in each experiment. A "baseline" experiment uses $L = \$3.60$, or an even cash/asset balance.

We consider the remaining eleven experiments, and obtain a single data point from each experiment so that a group of traders is not involved in more than one data point. In particular, we consider the average price in each experiment. We then have eleven independent observations, each involving a different group of people, to avoid issues of heteroscedasticity. These eleven average prices are 3.76, 3.73, 3.52, 4.33, 3.717 and 3.445 for the six cash-rich experiments (see Table 2), and 2.38, 3.04, 2.97, 2.84 and 2.89 for the five asset-rich experiments. Even the lowest average price in the cash-rich experiments is higher than the highest average price in the asset-rich experiments.

The cash-rich experiments have a mean of \$3.75 with a standard deviation of \$0.26, while the asset-rich experiments have a mean of \$2.83 with a standard deviation of \$0.31. The 95% confidence interval for the difference is (0.53,1.32). Testing for equal means using the t-test results in a strong statistical confirmation that the means differ, as one obtains $T = 5.37$, P = 0.0007 with degrees of freedom (DF) equal to 8.

We perform a non-parametric test on the medians of the two sets, \$3.73 for the cash-rich and \$2.90 for the asset-rich. The Mann–Whitney test (see Mendenhall, 1987 and Daniel, 1990) shows that the median of the cash-rich experiments is higher than the median of the asset-rich experiments, with a statistical significance of 0.0081. The 96.4% confidence interval for the difference is (0.54, 1.38). Hence, even when the most stringent statistical standards are used (e.g., relating to heteroscedasticity) there is a very strong statistical confirmation that the cash-rich experiments result in higher trading prices.

To understand the influence of liquidity on price throughout the experiment, we compute the correlation between price, $P(t)$, and liquidity, L, for each period separately, so that we have twelve independent observations for each of the fifteen periods. Table 3 shows the estimated correlation coefficient for each period. We can then test the sample correlation coefficients, r, displayed above for each period, as an estimator of the true coefficient, ρ. A test of the null hypothesis that no correlation exists between the price and liquidity, i.e., H_0: ρ. = 0, can be performed using the t distribution with $n = 12$ degrees of freedom. Defining $t = r(n - 2)^{1/2}(1 - r^2)^{-1/2}$, we find that the first seven periods satisfy $t > t_{0.05} = 1.82$, thereby establishing statistical significance at a 95% confidence level. During periods 3, 4 and 5, the 0.80 correlation with $n = 12$ leads to $t > t_{0.002} = 4.22$, establishing an extremely high probability that high liquidity is associated with high prices during the early periods.

In order to understand the extent to which liquidity influences price during different time periods, we estimate the rise in prices for each dollar of additional liquidity for each period. We use the linear prediction equation

$$Price(\tau, e) = \beta_{0,\tau} + \beta_{1,\tau}\, Liquidity(e)$$

where τ is the time period (1 through 15) and e is the experiment. Note that the liquidity value does not vary with the time period, but only with the experiment. Table 4 displays the values of β_0 and β_1 for each period, along with the values for the t-test and the P-values. The P-values are all below 0.002 during periods 2–5, and 0.01 or less in periods 2–7.

Thus, an increase of \$1 per share of extra cash in the market is associated with

1. A 29 cent increase in the average price per share during the first four periods;
2. A 19 cent increase during the middle periods (5–11);
3. An 11 cent increase during the final four periods (12–15).

As the experiment ends, the diminishing role of liquidity is replaced by the fundamental value (\$3.60) and culminates in a higher constant in the later periods, as indicated in Table 2.

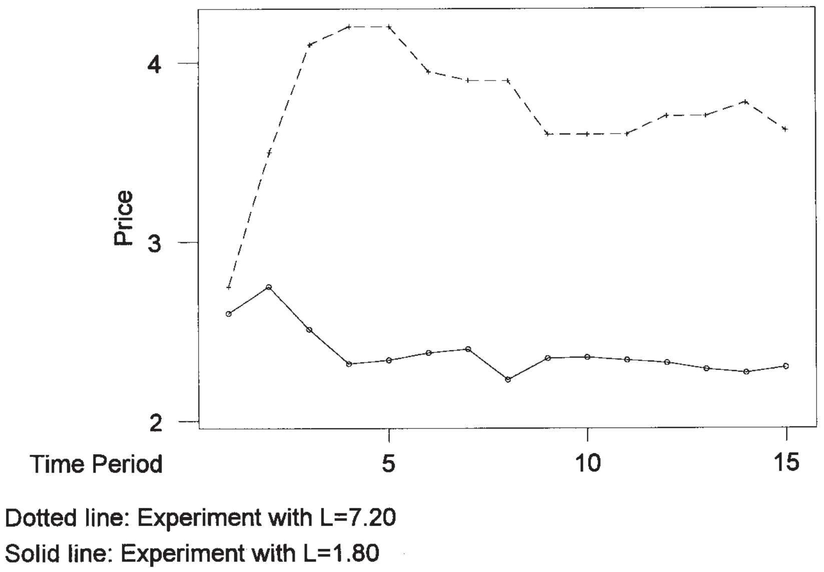
Thus the data indicate that the influence of liquidity is strongest during the first few periods after the first, and tends to diminish near the end when the proximity of the actual payout and the dwindling opportunity to trade the asset across time are apparent. With respect to all thirty-seven experiments reported here, we find on average that the maximum impact of the excess cash is not during the initial period, but during the second through fifth periods. The first period is unique in that no information about the other traders' strategies is available. During the second period, some information about others' strategies is available but no price change (i.e., momentum or trend) has emerged until the second period has ended. During the latter periods, traders know that the previous trading price reflects others'

Table 2. *The Constant Fund Value Experiments*

	L = 1.8	L = 7.2	L = 1.80	L = 7.20	L = 3.60	L = 4.68	L = 2.77	L = 4.68	L = 1.44	L = 5.40	L = 2.40	L = 3.96
Period 1	2.6	3.2	0.9	2.75	2.8	2.5	2.87	4.8	1.91	3.3	2.5	2.5
Period 2	2.75	3.5	1.32	3.5	3.2	3.04	3.1	4.5	1.97	3.875	2.75	2.8
Period 3	2.51	3.55	1.81	4.1	3.4	3.75	3.37	4.27	1.98	3.6	2.87	3
Period 4	2.32	3.7	2	4.2	3.5	3.5	3.62	4.49	1.995	3.75	2.95	3.41
Period 5	2.34	3.75	2.57	4.2	3.55	3.99	3.42	4.55	2.26	3.7	3.22	3.5
Period 6	2.38	3.8	3.5	3.95	3.5	3.5	3.37	4.6	2.4	3.87	3.2	3.8
Period 7	2.4	3.95	4.6	3.9	3.6	3.75	3.25	4.5	2.6	3.8	2.72	3.8
Period 8	2.23	4	5	3.9	3.8	3.51	2.5	4.3	2.805	3.88	2.8	3.56
Period 9	2.35	4.05	5	3.6	3.9	3.7	2.5	4.25	3.18	3.925	2.68	3.8
Period 10	2.355	4.1	3.3	3.6	3.95	3.6	2.52	4.4	3.43	3.925	2.8	3.8
Period 11	2.34	4.17	3.75	3.6	4	3.68	2.6	4.35	3.6	3.82	2.8	3.55
Period 12	2.325	4.24	3	3.7	4	3.6	2.72	4.31	3.7	3.66	3	3.65
Period 13	2.29	3.2	3.1	3.7	4	3.75	3	4.25	3.65	3.6	3.4	3.6
Period 14	2.27	4.11	3	3.775	4	3.54	3.16	3.845	3.6	3.5	2.77	3.6
Period 15	2.3	3.12	2.8	3.62	3.95	3.5	2.55	3.57	3.56	3.55	3	3.37
Exp Mean	2.384	3.7626667	3.0433333	3.7396667	3.6766667	3.5273333	2.97	4.3323333	2.8426667	3.717	2.8973333	3.4493333
Exp Med	2.34	3.8	3	3.7	3.8	3.6	3	4.35	2.805	3.75	2.8	3.56
Exp Max	2.75	4.24	5	4.2	4	3.99	3.62	4.8	3.7	3.925	3.4	3.8

Note: Twelve experiments (single bid-offer) with single payout of \$3.60 at the end differ only in terms of liquidity values, *L*. Prices are displayed for each of the 15 periods along with the mean, median and maximum of the prices during each experiment.

FIGURE 3
Price Evolution for Two Typical Constant Fundamental Value Experiments



Note: The price evolution for two of the experiments with single payout of $3.60 at the 15th period is shown. The dashed line shows that the time evolution when the liquidity value is L = $7.20 (twice as much asset as cash) is much higher than the reverse situation, L = $1.80.

opinions, as well as information on a price trend that may influence the momentum players.

One possibility is that some time scale is required for the effect of excess cash to translate into higher prices. In other words, a non-linear effect of excess cash is exhibited as traders first react to their own cash position, then implicitly take into account the cash position of others. For example, someone who places a buy order that is not accepted (because others with ample cash have outbid him) must consider whether to raise his bid the next time. Thus the explanation of the time scale required for the manifestation of excess cash may be related to the excess bids idea, as well as the momentum that is established as the excess cash leads to higher prices. This issue merits additional study to further separate the effects of undervaluation, momentum and excess cash.

Note that the initial trading price in these experiments is generally lower than in previous constant fundamental value experiments, such as those reported in Caginalp, Porter and Smith [1998]. One reason is that the average liquidity in the current set of experiments is lower than in the prior experiments, as none of the prior experiments used L = $1.80 or lower. The network program and instructions used in the two experiments also differ. The instructions in the former were longer (about one hour versus about one-half hour).

The initial price in most other experiments (including declining fundamental value experiments) has also been lower than fundamental value, and has exhibited considerable variation within a set of instructions. One reason for this general bias toward lower prices may be that participants (who have generally spent more time as consumers than sellers) are more experienced at seeking bargains than trying to establish higher prices (Miller, 2001). Our experiments indicate that part of the answer concerns the cash/asset ratio. There is a correlation of 0.51 between L and the period 1 price (as indicated in Table 3) with a t-test value of 1.87. The 0.8 correlation during periods 3, 4 and 5 emphasizes this relationship further. Earlier experiments are also compatible with this conclusion (Caginalp, Porter and Smith, 1998).

In summary, these experiments form the basis for a precise calibration of 1) the change in each period price as a function of the cash/asset ratio, and 2) the

Table 3. *Correlation Coefficients for IPO Experiments*

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Correlation	0.51	0.69	0.8	0.8	0.8	0.7	0.51	0.42	0.34	0.63	0.54	0.64	0.37	0.67	0.44
t-test	1.87	3.01	4.22	4.22	4.22	3.1	1.87	1.46	1.14	2.57	2.03	2.63	1.26	2.85	1.55

Note: For each of the 15 periods, one obtains 12 trading prices from the experiments. The correlation between price and liquidity value, L, is computed for each period using this statistically independent data. The prices are found to be highly correlated with the liquidity values, particularly for the early periods after the first. The t-test value is displayed below the correlation and indicates that price and liquidity are correlated within a statistical confidence of 95% for the first seven periods.

Table 4. *Mixed Effects Model Statistics for Beta0 and Beta1*

Period	Beta0	St Dev	t	p	Beta1	St Dev	t	p	F
1	1.8077	0.5367	3.36	0.007	0.2331	0.1235	1.89	0.089	3.56
2	1.8999	0.4154	4.57	0	0.28779	0.09546	3.01	0.013	9.09
3	1.9689	0.3243	6.07	0	0.31073	0.07451	4.17	0.002	17.39
4	2.002	0.3402	5.94	0	0.32373	0.07817	4.14	0.002	17.15
5	2.2959	0.2998	7.66	0	0.28765	0.0689	4.18	0.002	17.43
6	2.6341	0.3093	8.52	0	0.21863	0.07106	3.08	0.012	9.47
7	2.8683	0.418	6.86	0	0.18005	0.09605	1.87	0.09	3.51
8	2.8541	0.5027	5.68	0	0.1712	0.1155	1.48	0.169	2.2
9	3.0527	0.5031	6.08	0	0.1331	0.1156	1.15	0.276	1.33
10	2.6978	0.3424	7.88	0	0.20045	0.07869	2.55	0.029	6.49
11	2.8594	0.362	7.9	0	0.16934	0.08319	2.04	0.069	4.14
12	2.7259	0.3225	8.45	0	0.19592	0.0741	2.64	0.025	6.99
13	3.0872	0.3299	9.36	0	0.09575	0.0758	1.26	0.235	1.6
14	2.7198	0.275	9.89	0	0.18181	0.06319	2.88	0.016	8.28
15	2.8091	0.3039	9.24	0	0.11039	0.0683	1.58	0.145	2.5

Note: For each period, one computes the linear regression, $P(t) = \beta_0 + \beta_1 L$, using independent data from the 12 experiments. The data indicates that each dollar of additional liquidity results in about a 29 cent increase in trading prices during the early periods, a 19 cent increase during the middle periods and an 11 cent increase during the final periods. As the experiment nears its end, there is a shorter remaining time to trade, and a greater focus on the fundamental value, or the likely payout.

rate of convergence to equilibrium. They also provide a vehicle for understanding some of the problems related to initial public offerings (IPOs) and closed-end funds that have been noted by practitioners and academics. Many closed-end funds have traded at persistent discounts (see, for example, Lee, Shleifer and Thaler, 1993). From our perspective, it appears that the excess supply of shares compared to the available cash may be a primary reason for this chronic discount.

For example, underwriters planning to launch a fund that will invest in a particular country must consider the potential market (or the available cash) within the U.S. for investing in that country through this vehicle. If the available cash is, say, $200 million on the part of the public, while the initial market capitalization of the security is $300 million, the initial fundamental value of each of 10 million shares issued would be $30. The additional $100 million must be provided by the underwriters and additional institutions that would subsequently need to unwind their positions. However, the liquidity value would ultimately be $200 million/10 million shares = $20 per share. Of course, initially the $300 million must be available to purchase the stocks in the particular market. Once this is done, the total pool of cash is back to $200 million and the liquidity price is back at $20 per share, which is a 33% discount from the fundamental value of $30 per share of net asset value assuming no change in the underlying securities.

One feature of the IPO market that has attracted much attention relates to the rapid rise once trading begins. A possible rationale for this “underpricing” has been studied by Rock [1986], Chowdhry and Nanda [1996] and Kaserer and Kempf [1995].

Conclusion

The question of how rapidly prices approach equilibrium is a central issue in the development of a theory of price dynamics. The set of experiments with constant fundamental value (i.e., a single payout at the end) provides limited support for the efficient market hypothesis, since prices gradually approach fundamental value. The slow convergence toward this equilibrium as the payout period nears, however, indicates that the idealized game theoretic model is far from accurate. In particular, all aspects of the trading rules are known at the outset, and there is no additional information disclosed about the payout between periods 1 and 15. Consequently, any statistically significant price change is incompatible with classical game theory and any price theory that is built upon those assumptions. With no change in fundamental value, the temporal changes in price can only be based on the trading history during the experiment. On a more fundamental level, any change in price cannot be attributed to uncertainty about the expected payout and must therefore be related to the uncertainty about the actions of other traders (see the discussion of Smith, Suchanek and Williams, 1988).

Implications for Basic Price Theory

As noted in the Introduction, classical game theory is based on the hypothesis that agents not only self-optimize, but rely on the self-optimization of others. If traders relied on the self-optimization of others, who in turn do the same, then the initial trading price would be equal to the fundamental value. This is a consequence of game theoretic strategy, which would imply the lack of opportunity to sell above this price. As traders receive information (in the form of price evolution) on the motivations, strategy and psychology of the other traders, there may be an evolution toward greater reliance on others' self-optimization. As noted by Beard and Beil [1994], increased experience in the different roles of buyer and seller leads to greater reliance on others' self-optimization. This perspective on the approach to equilibrium leads to an explanation related to adaptive behavior and learning theory.

In all the constant fundamental value experiments, the trading price moves closer to fundamental value as traders receive more information about the trading patterns of others. Also, the large deviations from fundamental value at the outset may be attributed to the absence of information that traders have on others. A wide spectrum of bids and offers arises as a consequence. If there is an excess of cash, a small fraction of the bids balance the offers (which are more scarce due to the imbalance between cash and asset) and the trading cross occurs at a high price. At the end of the first period there is the same amount of cash in the system, but the price of the asset is much higher so that the ratio of cash to asset is much lower.

As a simple illustration, suppose there are ten shares and \$48 in cash at the start of the experiment. Since the only concept of price available for the shares at the start is the fundamental value, the cash-to-asset ratio can be regarded as 48/36 =1.33. Suppose the initial period trading price turns out to be \$7.20 at the end of the first period. Then the asset market value can be regarded as \$7.20 × 10 shares = \$72.00, so that the cash-to-asset ratio is 48/72 = 0.66. During the first period, there is enough cash to purchase thirteen shares above the fundamental value of \$3.60. However, during the second period, there is enough cash to purchase only six shares above the \$7.20 price. Thus, based on available cash, it is much more difficult to move prices higher once the price is at such a level without an additional infusion of cash. The number of shares is identical in both periods, so the ratio of available cash to asset price makes it more likely for prices to decline than to increase further. Once the price decline begins, momentum selling leads to a self-feeding mechanism that leads to lower prices.

The intricate relationship between momentum and liquidity may be the chief reason for the sudden changes that occur in the markets without any apparent rationale. The overvaluation of an asset, for example, may continue as an overreaction to some new information. A small trend that is thereby established leads to buying on the part of the momentum traders. This in turn leads to a more sustained trend that continues until the available cash is too small in comparison with the asset prices. The rally then runs out of steam and appears to turn abruptly and unpredictably without any new information on fundamentals.

This perspective is consistent with the observations in the experiments featuring declining fundamental value (classical bubble experiments). When there is a relatively small amount of cash endowed to the traders as a whole, the uptrend that starts due to undervaluation appears to be rather muted (see Figure 2). When there is ample cash, the momentum buying drives prices far higher than the fundamental value.

At a practical level, our findings are significant in resolving the causes of a market bubble, as well as determining the factors that can eliminate it. It has been noted that experience as a group is one of the few changes that can reduce the bubble (Smith, Suchanek and Williams, 1988, Porter and Smith, 1994). One limitation to the practical application of this result is that, in most markets, there are generally newcomers. We have presented two series of experiments indicating that three factors, applied in concert, can eliminate a laboratory bubble entirely: a low cash-to-asset ratio, dividends that are deferred and an open specialist's book. If all of these factors are reversed, we have a large bubble (with prices exceeding fundamental value by more than a factor of two).

In addition to providing some of the answers to the puzzle of market bubbles, these experiments highlight the importance of momentum and excess cash (liquidity). Furthermore, these features of trading imply certain prerequisites for any theory of price dynamics.

Liquidity, Momentum, and Fundamental Value

The complex interaction between liquidity, momentum and fundamental value suggests that time series forecasting is difficult beyond a narrow time horizon (Caginalp, Porter and Smith, 2000a). In particular, abrupt changes in price directions are generally not predictable with the statistical methods. However, a more fundamental modeling of the dynamics of trading and price movements could forecast such turning points if the interactions are properly modeled. The role of liquidity in volatility has also been noted (Peters, 1999).

A key advantage to this approach is that basic conservation laws can be used (e.g., total cash and number of shares are conserved) and economic behavioral hypotheses can be used, and the ideas of liquidity, momentum and fundamental value can be integrated in a cohesive way (Caginalp and Balenovich, 1994, 1999).

For example, one can begin with a standard (linear) model (Watson and Getz, 1981) stipulating that prices change in proportion to excess demand, and normalize with respect to supply. The demand and supply, in turn, depend on the fraction of total assets in the system in cash or stock, respectively, multiplied by the probability that a unit of cash or stock will be submitted for a trade.

In this way, one derives a set of differential equations (with parameters that can be calibrated with experiments or field data) with the potential to forecast an evolution of prices over a substantial time frame. The turning point in a bubble can be seen in terms of a critical point in the balance between the available cash/stock, fundamental value and the persistence of the uptrend.

In a subsequent paper we use the data obtained from these experiments to test the quantitative predictions of the equations.

Implications for World Markets and Securities Marketing

Earlier experiments using price collars for trading in the first period demonstrated that an initial undervaluation led to a larger bubble. This was one of the predictions of the differential equations models, and highlights the importance of momentum. For securities marketing and regulation in new markets, the creation of conditions that lead to substantial undervaluation may result in bubbles.

The effect of excess cash appears to be even more pronounced than momentum in terms of the magnitude of bubbles. The result that each extra dollar of cash per share leads to a dollar increase in maximum trading price has clear implications for both the underwriting of securities and monetary policy. In the frothy IPO market of the late 1990s, a key factor in the spectacular rise of some new issues is probably the imbalance between the small portion of the company sold to investors (i.e., the float, or the part of the shares not held by insiders) and the large appetite for these shares.

For example, a newly public company may have issued 10 million shares priced at $10, which would represent a “float” of $100 million. If the available cash for this issue is $1 billion, then the “liquidity” price, L, within our perspective, would be $1 billion divided by 10 million shares, or $100 per share. Thus, share prices would tend to trade near $100. However, a large portion of the shares, owned by insiders, is generally “locked up” for six months under securities regulations and cannot be sold. If the fraction owned by insiders is 95% of the shares, then the market value of the hypothetical company is 20 × 10 million shares × $100/share, or $20 billion! This is a simple explanation for the extreme market valuations in which companies with no revenue, let alone earnings, vaulted past some of the largest American companies within days of becoming public. This explanation effectively reduces the problem of valuation to a problem of understanding the basic psychological and sociological reasons, in addition to the economic, for the funneling of large sums into the select segments of the marketplace.

In terms of world markets, the experiments suggest that the “easy money” policies of central banks lead to higher prices in financial markets. Economists often regard a nation’s stock market as a barometer of the strength of the economy, so a rising market is considered a good omen. However, from our experimental perspective, a rising market and high valuations may signify an overly relaxed monetary policy, in which assets (rather than common goods) are becoming inflated and pose a boom–bust threat.

It is generally acknowledged that central banks should not attempt to influence stock market prices, for doing so would defeat the purpose of a free market. Yet, from our perspective, the actions of central banks have a profound influence on the price levels of markets. The expansion of price/earnings ratios in U.S. stocks during the mid-1990s may have been enhanced by the Federal Reserve’s easing of monetary policy in response to the savings and loan crisis. Similarly, the Fed’s easing of interest rates during the fall of 1998, this time in response to the insolvency of Long Term Capital Management, and the precautionary increase in liquidity in anticipation of a Year 2000 problem, occurred during a time of economic expansion and may have contributed to the bubble of 1999.

Within a complex economy, many factors may also influence the level of excess cash available for investments. Government policies, demographic changes and a changing economy can shift more of the wealth to age and income groups that are more likely to invest the money than spend it. During the late 1990s, several factors provided additional cash for the market: 1) baby boomers reached their prime earning years and accumulated money that was available for investment, particularly through retirement accounts; 2) the computer revolution liberated a considerable amount of cash for corporations because technological advances meant they needed fewer employees; 3) tax policies of the 1980s that were favorable to the affluent resulted in more cash in the hands of people who were more likely to invest than to spend the additional income.

Even during the great tulip bubble in Holland, analogous factors related to liquidity were relevant. Holland was just emerging from a depression and an episode of Bubonic plague, so wages were rising for survivors. Trading became standardized as regular meetings were set up for transactions. An early version of margin trading emerged as participants were able to buy a share of a tulip bulb without payment of the entire sum (Dash, 1999).

This perspective thus indicates that one should be cautious about the standard inference that a rising stock market indicates a healthy economy. Often, the liquidity factors that create a rising market also provide fuel for an economic boom. This is not always the case, however, as the market bubble of Japan in the 1980s showed. While the Japanese market soared, the inefficiencies of the economy did not allow the excess cash to be invested toward a sustainable economic boom. A similar reasoning applies to a particular sector of a market. While the excess cash that flowed into the high-tech sector of the U.S. market in the late 1990s led to soaring prices, a sustained boom was not possible for many of the new Internet-related companies due to their underlying business structures.

In summary, stock and other asset prices are influenced by factors beyond the market's realistic assessment of value. The level of cash available for investment in a particular type of investment appears to be chief among them.

Acknowledgments

The authors are grateful for support from the Fred Maytag Family Foundation and the Dreman Foundation. Discussions with Dr. Mark Olson are also much appreciated.

References

Beard, T., and R. Beil. "Do People Rely on the Self-Interested Maximization of Others? An Experimental Test." *Management Science,* 40 (1994), pp. 252–262.

Caginalp, G., and D. Balenovich. "Asset Flow and Momentum: Deterministic and Stochastic Equations." *Phil. Trans. Royal Soc. London* A, 357 (1999), pp. 2119–2133.

Caginalp, G., and D. Balenovich. "Market Oscillations Induced by the Competition between Value-Based and Trend-Based Investment Strategies." *Applied Mathematical Finance,* 1 (1994), pp. 129–164.

Caginalp, G., D. Porter, and V. Smith. "Initial Cash/Asset Ratio and Asset Prices: An Experimental Study." *Proceedings of the National Academy of Sciences,* 95 (1998), pp. 756–761.

Caginalp, G., D. Porter, and V. Smith. "Momentum and Overreaction in Experimental Asset Markets." *Int. J. Ind. Org.,* 18 (2000), pp. 187–204.

Caginalp, G., D. Porter, and V. Smith. "Overreactions, Momentum, Liquidity and Price Bubbles in Laboratory and Field Asset Markets." *Journal of Psychology and Financial Markets,* 1 (2000), pp. 24–48.

Chowdhry, B., and V. Nanda. "Stabilization, Syndication and Pricing of IPO's." *Journal of Financial and Quantitative Analysis,* March 1996.

Daniel, W. *Applied Nonparametric Statistics,* 2nd ed. Boston: PWS-Kent, 1990.

Dash, M. *Tulip Mania.* New York: Random House, 1999.

Davis, D.D., and C.A. Holt. *Experimental Economics,* Section 3.7. Princeton, NJ: Princeton University Press, 1993.

Dreman, D. *Contrarian Investment Strategies, The Next Generation.* New York: Simon & Schuster, 1998.

Dreman, D., and E. Lufkin. "Investor Overreaction: Evidence that Its Basis is Psychological." *Journal of Psychology and Financial Markets,* 1 (2000), pp. 61–75.

Kaserer, C., and V. Kempf. "Das Underpricing-Phanomen am deutschen Kapitalmarkt und seine Ursachen—Eine empirische Analyse fur den Zeitraum 1983–1992." *Zeitschrift fur Bankrecht und Bankwirtshaft,* 1 (1995), pp. 45–68.

Lee, C., A. Shleifer, and R. Thaler. "Investor Sentiment and the Closed-End Fund Puzzle." *Journal of Finance,* 48 (1993), pp. 75–109.

Lei, V., C.N. Noussair, and C.R. Plott. "Non-Speculative Bubbles in Experimental Asset Markets: Lack of Common Knowledge of Rationality vs. Actual Irrationality." Preprint, 1998.

Mendenhall, W. *Introduction to Probability and Statistics,* 7th ed. Boston: Prindle, Weber and Schmidt, 1987.

Miller, R. *Paving Wall Street: Experimental Economics and the Quest for the Perfect Market.* New York: John Wiley & Sons, Forthcoming, 2001.

Peters, E. *Fractal Market Analysis.* New York: John Wiley & Sons, 1994.

Porter, D., and V. Smith. "Stock Market Bubbles in the Laboratory." *Applied Mathematical Finance,* 1 (1994), pp. 111–128.

Rock, K. F. "Why New Issues are Underpriced." *Journal of Financial Economics,* 15 (1986), pp. 187–212.

Shiller, R.J. *Irrational Exuberance.* Princeton, NJ: Princeton University Press, 2000.

Smith, V.L. "Microeconomic Systems as an Experimental Science." *American Economic Review,* 72 (1982), pp. 923–955.

Smith, V.L., G.L. Suchanek, and A.W. Williams. "Bubbles, Crashes and Endogenous Expectations in Experimental Spot Asset Markets." *Econometrica,* 56 (1988), pp. 1119–1151.

Van Boening, M.V., A.W. Williams, and S. LaMaster. "Price Bubbles and Crashes in Experimental Call Markets." *Economics Letters,* 41 (1993), pp. 179–185.

Watson, D.S., and M. Getz. *Price Theory and Its Uses,* 5th ed., Lanham, MD: Houghton Mifflin, 1981.

Appendix

Instructions for Participants in Experiments

The following instructions were displayed on the computer terminal for the participants:

This is an experiment in market decision-making, and you will be paid for your participation in cash at the end of the experiment. Different participants may earn different amounts. What you earn depends on your decisions and the decisions of others.

The experiment will take place through computer terminals at which you are seated, and interaction among participants will take place primarily though these computers. It is important that you do not talk or in any way try to communicate with other participants during the experiment. If you disobey the rules, we will have to ask you to leave the experiment.

We will start with a detailed instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to interact with the computers.

If you have any questions during the instruction period, raise your hand and your question will be answered so everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and a monitor will come and assist you.

In this experiment, you will be trading an "Asset" which we will label as Asset A. This asset will live for EXACTLY 15 periods of trading. After each trading period the asset will earn a dividend which will equal 0, 8, 28, or 60 cents. Each of these dividend amounts are equally likely so that on average, over many draws, the dividend would be 24 cents each period. Thus if you had a unit of the asset at the end of period 1 it would return you an average of 24 cents. Since the experiment lasts 15 periods, in which these draws are made after each period, if you held an asset from period 1 until the end of the 15th period, that asset would return to you on average a total of **$3.60 (15 times 24 cents)** over the 15 periods. Similarly, if you bought a unit of the asset in period 2 and held it from period 2 until the 15th period, the average accumulated dividends would be **$3.36 (14 times 24 cents).** The table on the left shows the average holding value of the asset if it is held from the listed period until the last period of trading. We will now describe the mechanism you will be using to make trades and the accounting system that keeps track of your earnings and trades.

All communication during the experiment will take place through your keyboard and mouse. All results from each round are relayed to you through your screen. We will now introduce you to the screen displays you will be using and how to make bids and read the results.

Your order book is broken down into two large display areas labeled **Bid Book** and **Results.** On the very top of the page you will find listed your **ID** number, the Trading **Period**, and the **Time** in seconds left in the trading period.

Your Bid Book contains information about your offers in the market and your current asset and cash positions. Under the heading **Asset Information,** you will find a message about the value structure of the asset. Below that you will find the current level of asset that you own. In this case you have 3 units of the Asset that you can keep or sell. At the bottom part of the left-hand side of your screen, we list your current level of **Available Cash** that you can use to try and purchase units of the asset in the market. In this case you have 720 cents in cash at this moment.

On the Offer Book side of the Bid Book screen, you will find market information. This is where you can submit BIDS to BUY units of the asset and ASKS to SELL units of the asset. Let us go through some examples to see how this market works. At the beginning of each market period, you will have 300 seconds to submit Sealed Offers (you are the only one who sees your bids and asks when you submit them).

On the Offer Book side of the Bid Book screen, you will find market information. This is where you can submit BIDS to BUY units of the asset and ASKS to SELL units of the asset. Let us go through some examples to see how this market works. At the beginning of each market period, you will have 240 seconds to submit Offers to the market.

Let us first begin by creating a BID to BUY 2 units of asset A for 50 cents *Per Unit*. Please submit this order by typing 50 in the **price box** of the Bid portion of your order book and 2 in the **units box** (the titles are in *red*). After you have done this press submit to see what happens.

This bid is now listed in your order book with an identifying number 1. Your available cash to make bids with is now down by 100 cents (50 times 2 units). You have provisionally committed to this order. Go ahead and place another bid for 1 unit at 60 cents. Please do this now.

This bid is now listed in your order book with an identifying number 2. Your available cash to make bids with is now down by the 60 cents you have provisionally committed to this order. The two orders represent your willingness to pay up to 60 cents for one unit of A and up to 50 cents per unit for as many as 2 units of A.

Let us now submit some asks to the market. To do this go to the ask portion of your Order Book and submit an ASK to SELL 1 unit of A for 100 cents. To do this place 100 in the **price box** and 1 in the **units box** and press submit in the ask part of the book. Please do this now.

This ask is now listed in your order book with an identifying number 1. Your current inventory available to make asks is now down to 2 since you have provisionally committed a unit in your ask. Go ahead and place another ask for 2 units at 50 cents per unit. Please do this now.

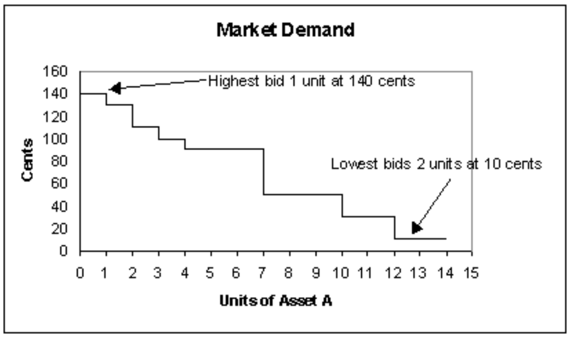
Your current inventory that you can commit to selling into the market is now 0 and you have submitted two asks to the market, which represent your willingness to sell 1 unit for at least 100 cents and sell up to 2 units at a minimum price of 50 cents each. Before time runs out in the market you can submit new bids and asks, change or Edit a submitted bid or ask, or remove a submitted bid or ask. Let us go through some examples to illustrate these features.

Suppose you would like to remove your ask #2 which has 2 units at 50 cents each. To do this click in the box where Ask #2 resides and then click on Remove. Please do this now.

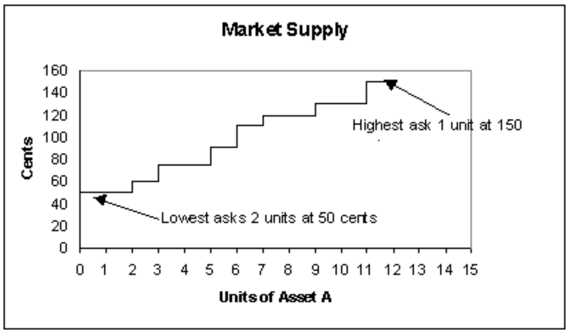
Your Ask #2 is now removed and your available inventory has gone up to 2 units. Suppose you would like to change or Edit your Bid #2 from 60 cents to 100 cents. To do this click in the box where Bid #2 resides and then click on Edit and change the bid from 60 to 100. Please do this now.

Your Bid #2 has been changed and your available cash has gone down by 40 cents. Once the time in the trading period is expired, all the bids and asks submitted by all traders will be sent to the central market to determine who traded and at what prices. We will describe the rule we will use to select traders and the trade price next.

To determine the Market Demand for the asset, we first take all the bids and rank them from highest bid price to lowest bid price and graph them. To the right you will find an example Market Demand curve derived from the bids to buy units of the asset. In this graph, we see that the highest bid submitted was for 1 unit at 140 cents. The lowest bids are for units 13 and 14 for 10 cents. These bids have been pointed out in this graph.



To determine the Market Supply for the asset, we first take all the asks and rank them from lowest ask price to highest ask price and graph them. To the right, you will find an example Market Supply curve derived from asks to sell units of the asset. In this graph we see that the lowest ask submitted was for 2 units at 50 cents. The highest ask was for unit 12 for 150 cents. These asks have been pointed out in this graph.



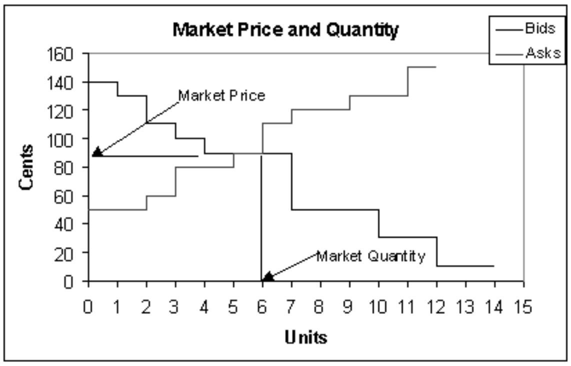
As asks enter the market, the Market Supply Curve is updated. You can find the Market Supply Curve in red below. In the graph below, we see that the lowest ask submitted was for 2 units at 50 cents. The highest ask was for unit 12 for 150 cents. These asks have been pointed out in this graph.

After we have established the Market Demand and Supply curves, we put them together to find the trades and price. The number of units traded is where the curves intersect. The price is also determined where the curves intersect. In the graph to the right, we see that the curves intersect at 6 units at 90 cents. This means that all those who submitted bids at prices higher than 90 cents have traded and will pay LESS THAN what they bid since the price is 90 cents. In addition, all those who submitted asks below 90 cents have traded and will receive MORE THAN what they asked since the market price is 90 cents. Those who bid below the market price do not trade and those who asked above the market price do not trade. Those who bid and asked at the market price may or may not trade since rationing may occur to make sure supply equals demand.

As bids and asks arrive in the period, the Market Demand and Supply curves are put together to show the TENTATIVE trades and market price. This condition is labeled TENTATIVE because as new bids and asks arrive to the market the curves will change and trades and price may move. The number of units traded is where the curves intersect. The price is also determined where the curves intersect. In the graph to the right, we see that the curves intersect at 6 units at 90 cents. The price and number of units traded will be final when the market closes at the end of the period, which lasts 240 seconds.

Once the market cross is found you will be taken to the results tab on your Market screen. Please click the results tab to see what they look like.

As we mentioned above, as bids and asks arrive to the market, the Market Demand and Supply curves are updated. To see how this updating process is done look at the graph below which shows how the curves update along with price and trade volume as bids and asks arrive to the market.



On the left-hand side of the results page we list the dividend draw for the period, which in this example is 28 cents. The accounting sheet below has your cash accounting, which starts with your initial cash position for the period of 720. We add to that dividends from your asset (you have 4 units times 28 cents = 112). Next, we add any revenue from sales of your assets, in this case you made no sales so that is left at 0 and subtract from that money you spent to buy assets (90 cents in this case). When we add up all those amounts, we find your cash available for the next period is 742.

The market cross and trade information is found on the right-hand part of your screen.

To go on to the next period, you need to click the ready button at the top of the screen. When everyone has clicked ready, the next period will start. Once the next market period starts, your asset and cash balance will be updated to reflect your trades and dividend accumulation. Please click Ready now to see what they look like.

At the beginning of Period 2, you now have 4 units of the asset you can sell or hold, and you have 742 cents that you can use to make bids or hold. We will do this for 15 periods. At the end of period 15, there will be the last dividend draw, which will be as before either 0, 8, 28, or 60 cents. After that draw, your cash position will be calculated and you will be paid that amount. Other than the final dividend draw, the asset will be worth nothing.

Summary

1. You will be given a starting amount of cash and units of Asset A.
2. Asset A generates a dividend of 0, 8, 28, or 60 cents at the end of each of 15 trading periods. Each dividend outcome is equally likely to be picked at the end of each period. Thus, the average dividend per period is 24 cents.
3. You can submit BIDS to BUY units of the asset and ASKS to SELL units of the asset.
4. As bids and asks arrive to the market during a trading period, the Market Demand and Supply curves are updated. The TENTATIVE market trading price and quantity are determined by where the Market Demand Array crosses the Market Supply Array.
5. When the market period closes at the end of 240 seconds, BIDS above the market price are accepted and ASKS below the market price are accepted. BIDS below and ASKS above the market price are not accepted.
6. The market lasts for 15 periods. At the end of period 15, there will be one last dividend of 0, 8, 28, or 60 cents. After that the asset expires and is worth nothing to you.

To go onto the review quiz, please click Next.

Review Quiz

1. How many trading periods does this experiment last?
2. At the end of each period, the asset earns a dividend of:
3. If you had a unit of the asset in period 3, and you held it until period 15, what would be the *average* amount of dividend it earned?
4. You can submit asks to the market to:
5. Prices in your bids and asks are stated as:
6. If your bid is accepted, the market price will be:
7. If your ask is accepted, the market price will be:

To start the experiment, please click Start. To review the instructions, please click Back. Once you click Start, you will not be able to return to the instructions.

Statistical Analysis

We present below the statistical analysis that augments the results of the section "Bubble Experiments (Declining Fundamental Value) with Varying Conditions."

A linear regression analysis for the maximum positive deviation from the fundamental value yields the following regression equation with positive coefficients for each of the variables:

$$MaxDevPrice = -50.9 + 70.7\ Liquidity + 13.7\ DivDistr + 37.5\ ClosedBk$$

Once again, there is extremely strong statistical evidence for the hypothesis that excess cash produces a larger bubble (P-value of less than 1/10,000), and some weak evidence ($T = 0.69$ and $P = 0.50$) that the closed book variable results in a larger bubble. The analysis of variance results in an F-value of 12.8.

As noted in the Introduction, the differential equations models suggest that a low initial price should result in a larger bubble. Using this as an additional predictor in the regression analysis for the maximum positive deviation from fundamental value, we obtain

$$MaxDevPrice = 15 + 77.2\ Liquidity + 21.7\ DivDistr + 17.7\ ClosedBk - 0.97\ InitialPrice$$

The coefficient of –0.97 for the initial condition has a T-value of –0.62, with $P = 0.539$ and $F = 9.4$. These results, although not the focus of this study, are consistent with earlier findings. In particular, an initial undervaluation tends to draw buying from traders focusing on fundamentals. A lower initial price leads to stronger buying, which the momentum traders find more attractive, which leads to a stronger self-feeding mechanism and a larger maximum deviation from fundamental value.

Examining the maximum value of the trading price for each experiment, we find the linear regression:

$$MaxPrice = 168 + 76.2\ Liquidity + 52.9\ DivDistr + 5.6\ ClosedBk - 1.31\ InitialPrice$$

The coefficient of the L variable has a T-value of 5.34 with $P < 1/10{,}000$. The coefficient of the dividend variable has a T-value of 0.88 with $P = 0.387$. The closed book variable has a coefficient with a T-value of 0.10 with $P = 0.92$. The initial price coefficient corresponds to a T-value of –0.99 with $P = 0.34$.

Next, we examine the statistical difference among the CR/DP/CB groups of experiments favoring higher prices and larger bubbles, versus the AR/DD/OB groups favoring lower prices and smaller bubbles.

The mean of the average trading price of each of the experiments in the CR/DP/CB group is 384.3, with a standard deviation of 59.7, while the mean of the AR/DD/OB group is 103.5 with a standard deviation of 34.5. We perform a two-sample t-test and find that a 95% confidence interval in the difference of the two means is (154, 407). The hypothesis that the two sets of average prices differ is confirmed statistically, with a T-value of 7.05 and $P = 0.0059$ ($DF = 3$).

We perform a similar test to compare the two sets in terms of the maximum prices in each experiment. We find that the CR/DP/CB group has a mean maximum value of 760 (standard deviation of 139) compared with 162 (standard deviation of 97.8) for the AR/DD/OB group. The 95% confidence interval for the difference is (286, 910). The hypothesis that the two sets of maximum prices differ is confirmed statistically, with a T-value of 6.09 and $P = 0.0059$ ($DF = 3$).

Finally, we examine the maximum price deviation from the fundamental value for the two sets of experiments, which have an average of 672 (standard deviation of 148) and 32.3 (standard deviation of 23.3), respectively. The 95% confidence interval for the difference is (268,1011). Once again there is a statistical confirmation that the two sets of maximum deviations differ at the level of $T = 7.40$ and $P = 0.018$ with $DF = 2$.

Similarly, we perform a set of non-parametric tests using the Mann–Whitney procedure to establish a statistically significant difference in the medians of two sets of numbers (see, for example, Mendenhall, 1987 or Daniel, 1990). A test of the average trading price of each experiment results in medians of 373.3 and 83.7, respectively, with a 91.7% confidence interval for the difference of (187, 365) with $W = 15$. The hypothesis that the medians are unequal is confirmed at the statistical level of $P = 0.08$. For the maximum trading prices the medians are 830 and 112. The 91.7% confidence interval is (325, 750) with $W = 15$. The hypothesis that the medians differ is confirmed at the level of $P = 0.08$.

Finally, in comparing the medians of the maximum deviations from the fundamental value, we find 730 versus 22 for the two sets of experiments. The 91.7% confidence interval for the difference is (445.1, 765.9) with $W = 15$. The hypothesis that the maximum deviation from fundamental value differs in the two sets is statistically confirmed at $P = 0.08$.

Next we consider subsets of the data, beginning with the closed book and dividends paid case, which are characteristic of a classical bubble experiment. For these ten experiments we perform a linear regression on the mean against the only remaining variable, the initial cash/asset ratio L, to obtain

$$MeanPrice = 51.7 + 44.9\ Liquidity$$

The standard deviation of the coefficient of L is 5.5, resulting in a T-value of 8.14 and $P < 1/10{,}000$. The F-value is 66.32.

Similarly, a linear regression for the maximum price in terms of the initial cash/asset ratio yields

$$MaxPrice = 19.4 + 99.9\ Liquidity$$

The standard deviation of the coefficient of L is 12.8, resulting in a T-value of 7.8 with $P < 1/10{,}000$ and an F-value of 60.5.

The linear regression for the maximum deviation from the fundamental value in terms of the cash/asset ratio yields

$$MaxDevPrice = -164 + 111\ Liquidity$$

The standard deviation of the coefficient of L is 17.4, resulting in a T-value of 6.36 with $P < 1/10{,}000$ and an F-value of 40.5.

We consider the same issue under open book conditions (with dividends paid each period as before). The three linear regressions for the mean price, the maximum price and the maximum deviation from fundamental value are

$$MeanPrice = 130 + 27.9\ Liquidity$$

(the standard deviation of the coefficient of L is 5.4, with $T = 5.2$, $P = 0.02$ and $F = 27$);

$$MaxPrice = 279 + 35.5\ Liquidity$$

(the standard deviation of the coefficient of L is 13.4, with $T = 2.7$, $P = 0.04$ and $F = 7.01$);

$$MaxDevPrice = 155 + 31.6\ Liquidity$$

(the standard deviation of the coefficient of L is 12.8, with $T = 2.5$, $P = 0.05$ and $F = 6.10$).