Wilson's Case Against the Dome: Not Necessary, Not Sufficient

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February 12, 2009

Abstract

Mark Wilson (2008) has recently located the pathology of Norton's (2006) dome in the assumption that we can consistently decompose Newtonian forces into 'reactive' and 'constraint' categories. It is shown here that this decomposition is neither necessary nor sufficient for Norton-style indeterminism.

1 Wilson's Case

Mark Wilson has recently argued that, "Any friend of determinism should be cautious about allowing forces to be glibly divided into 'reactive' and 'constraint' categories, for that's how Norton's loss of determinacy secretly enters the scene" (Wilson 2008, 6). In this discussion, I first show that the "glib division" identified by Wilson is not necessary for Norton-style indeterminism[1]. I will then show that it is not sufficient, either. So, although Wilson has presented us with an interesting difficulty in reconciling our idealizations in Newtonian physics, he has not managed to put his finger on the pathology behind the dome.

Wilson begins by identifying three conceptual frameworks for classical Newtonian physics, dubbed *mass point particle mechanics* (MP), the *physics of rigid bodies and perfect constraints*

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[1]Hereafter, I will identify 'Norton-style indeterminism' with the failure of a system to admit a unique Cauchy evolution, stemming from the failure of its equation of motion to satisfy the Lipschitz condition.

(PC), and *continuum mechanics* (CM). His very interesting thesis is that no combination of these frameworks can fill all the 'explanatory gaps' in our physical theorizing. The upshot, according to Wilson, is this:

> As long as such gaps persist, Norton-like indeterminacies may sometimes creep in, largely as a consequence of having adopted some fill-in 'rule of thumb' (e.g., the constraint provided by Norton's perfectly rigid track), which we don't believe truly 'gets all of the classical physics of the real-life situation right.' Accusations of 'indeterminism' rarely seem definitive in such cases, simply because we've never really trusted the 'rules of thumb' upon which they trade in the first case.

The offending 'rule of thumb,' we are told, is the one by which (in the PC framework) constant gravitational force is decomposed into perpendicular (constraint) and tangential (active) components along a surface. The worry is that, "from the points of view of our alternative foundational starting points [such as MP or CM], this kind of 'active/constraint' decomposition may prove strictly unwarranted and can only be justified as a form of convenient approximation" (Wilson 2008, 5). Wilson's point is that by ignoring MP phenomena such as jittery molecular interactions, the decomposition rule fails to adequately describe a classical Newtonian system.

As a consequence, Wilson says, "the apparent failures of determinism enter largely as an artifact of the fact that the 'rules of thumb' commonly cited in repair are often unable to plug the descriptive holes thoroughly" (Wilson 2008, 11). This is a very attractive and perhaps ultimately correct suggestion. However, the question for us is whether or not the "rule of thumb" that Wilson has identified is actually tied to the "apparent failures of determinism" in classical physics. I will now show that it is not.

2 Not Necessary, Not Sufficient

Consider a hemisphere with a mass at the apex. This system betrays exactly the same descriptive inadequacy that Wilson imputes to Norton's dome. A mass sitting at the apex is certainly not an idealization that can withstand the subtle intermolecular forces that Wilson entertains. So the decomposition of forces into perpendicular and tangential components is presumably no more licit. However, a hemisphere is a C^∞ surface, and so the usual uniqueness theorem for ordinary differential equations holds everywhere on it. So clearly, this component-wise decomposition is not sufficient for Norton-style indeterminism in classical physics.

As it turns out, component-wise decomposition of forces isn't necessary for Norton-style indeterminism, either. The easiest way to see this is to place the dome in a 'funny' force field[2] that is always tangent to the surface, characterized by the equation,

$$F = r^{1/2}.$$

As in Norton (2006), this r is a coordinate system set on the surface of the dome with the origin at the apex. Such an arrangement gives rise to the same equation of motion as Norton derived for his dome, $\ddot{r} = r^{1/2}$. Consequently, this system exhibits the same degree of indeterminism as Norton's original dome. And yet, there is no decomposition of forces, since the perpendicular 'constraint' component has been defined to be zero everywhere.

Lest such a force field be written off as arbitrary, here is a (perhaps) more interesting Newtonian arrangement to illustrate the point. The system I will describe involves no surface at all[3]. We will obtain a mass on a spring (Figure 1) for which there are two possible evolutions: one in which the mass just sits there for all of time, and another in which it starts bouncing for no apparent reason.

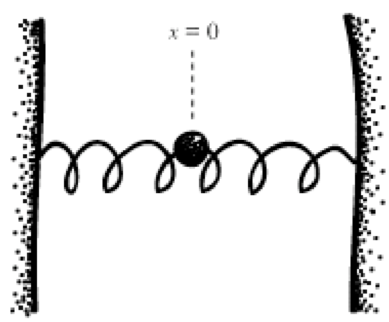

Figure 1: A mass and spring oscillator.

Take a spring with a mass in the middle that's sitting at rest. Specify the center of a coor-

[2] A similar arrangement was observed by Malament (2007).

[3] Zimba (2008) has also described a surface-less example of Norton-style indeterminism. However, Zimba's example is in electromagnetism, and our concern here is with classical Newtonian mechanics.

dinate system x with the origin at the center of the mass. The differential equation describing the motion of this system (in a neighborhood of the origin) is that of a harmonic oscillator,

$$\ddot{x} = -k^2 x,$$

where k is a constant describing the tension of the spring. Now, imagine that we wind up the spring very tightly, and then start letting it loose again, at a rate determined by the equation,

$$-k^2 = \frac{6}{t^2}. \tag{1}$$

Then the equation of motion for this spring system is given by,

$$\ddot{x} = \frac{6x}{t^2}.$$

One obvious solution to this differential equation is $x(t) = 0$, for all time t. This represents an evolution in which the mass just sits there at rest. However, it is easily shown[4] that there is also a second solution:

$$x(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ t^3 & \text{if } t \geq 0. \end{cases}$$

So, determinism fails for this spring system: there is more than one Cauchy evolution for the initial conditions, $x = 0$, $t \leq 0$. In one case, the mass remains at rest, and in the other, the mass starts bouncing after time $t = 0$ (Figure 2).

The example is meant to illustrate how Norton-style indeterminism can indeed appear in the absence of any component-wise decomposition of forces on a surface. It arises here in the absence of any surface at all.

Of course, there are several pathologies in this system, so let me just point out what seems to be the main one: the spring becomes 'infinitely tight' at time $t = 0$, as is visible in Equation (1). It's certainly fair to note that no real spring can stand to be wound infinitely tight. One might even say, following Wilson's lead, that such an account of intermolecular forces is ruled out by

[4]If $x = t^3$, then by taking two derivates we get that $\ddot{x} = 6t$. By substitution, this implies that $\ddot{x} = 6x/t^2$, as required.

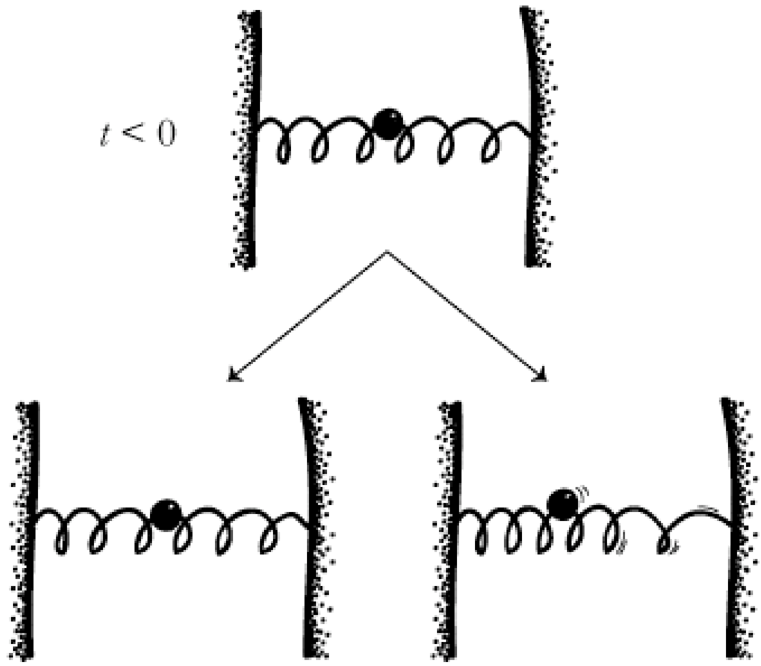

Figure 2: Two possible evolutions: one in which the mass remains at rest, and the other in which it starts bouncing for no apparent reason.

any good MP framework for Newtonian physics. But, even granting all this, my point remains: the "glib division" of forces into perpendicular and tangential components is not necessary for Norton-style indeterminism.

3 Conclusion

I take Wilson's overarching thesis in this paper to be of great interest: classical physics appears to be a beast with many heads, none of which play very well together. However, his analysis of the dome in this context misses the mark. We might agree that the dome is somehow pathological. But this pathology does not lie in the idealized decomposition of forces.

References

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