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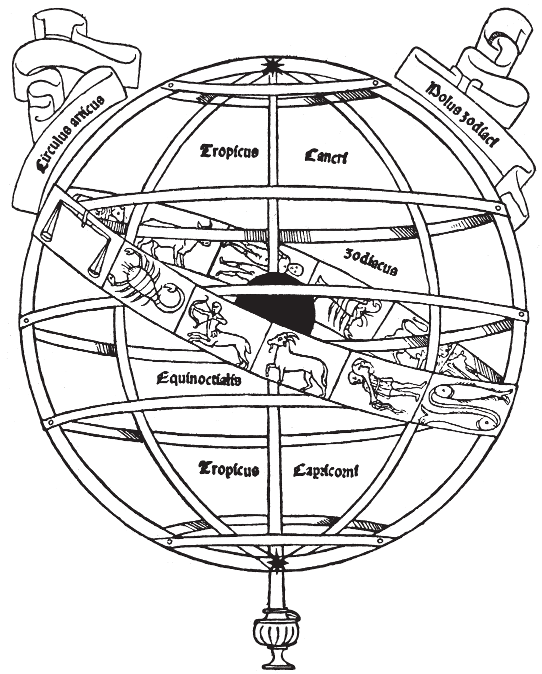

LEO S. OLSCHKI EDITORE
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APOLLONIUS OF PERGA'S CONTRIBUTIONS TO ASTRONOMY RECONSIDERED

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ABSTRACT — Apollonius of Perga (*ca*. 200 B.C.) is generally credited with having a critical role in the development of ancient planetary astronomy, both in respect to the equivalence of the eccentric and epicyclic models as well as in respect to the theorem on planetary stations and retrogradations. Doubt will be cast on the evidence used to support both claims. Moreover, even if the historicity of these claims is maintained, it is not clear that Apollonius had a significant impact on the development of astronomy before Ptolemy (*ca*. 150 A.D.). In particular, the evidence for awareness of Apollonius's theoretical achievements by Hipparchus (*ca*. 130 B.C.) – the most important astronomer between Apollonius and Ptolemy – will be shown to be weak.

1. Introduction

Apollonius of Perga (*ca.* 200 B.C.) was certainly one of the outstanding mathematicians of Antiquity.[1] But there is no extant treatise by Apollonius on astronomy, and his reputation in this regard depends on references in later works. The questions to be addressed here primarily concern claims for Apollonius's contributions to planetary theory. In *Almagest* XII.1 Ptolemy (*ca.* 150 A.D.) ascribes the theorem on stationary points to

* I am grateful to Giora Hon and Len Berggren for their comments on a draft of this paper.

[1] For Apollonius's life and his contributions to mathematics (notably conic sections) see, e.g., G.J. Toomer, *Apollonius of Perga*, in *Dictionary of Scientific Biography*, ed. by C.C. Gillespie, New York, Scribner, 1970-1980, vol. I, pp. 179-193.

Apollonius: is this claim credible? It is assumed, largely on the basis of this theorem, that Apollonius knew the equivalence of the eccentric model and the epicyclic model (*Almagest* III.3 for the Sun, and IV.5 for the Moon) even though no text between the time of Apollonius and Ptolemy makes this claim. The second claim need not depend on the first but, in that case, it is essential to demonstrate that Hipparchus (*ca.* 130 B.C.) was aware of this equivalence although it is certainly not sufficient to support the claim on behalf of Apollonius. The reason why the role of Hipparchus is so important is that, if Hipparchus was unaware of this equivalence, it is hard to see the work of Apollonius as having a significant impact on astronomers before Ptolemy. In this article doubt will be cast on Apollonius's contributions to planetary astronomy, and Hipparchus's knowledge of the equivalence of the two models (or hypotheses) will be shown to be weakly supported in the *Almagest*. No relevant works of Hipparchus survive.

2. Historical Background

The data for the contributions to astronomy by Apollonius are summarized by Neugebauer:

> We have from antiquity several short references to astronomical investigations of Apollonius of Perga, and all of them specifically refer to the moon. Ptolemaeus Chennus (about 100 A.D.) says (as quoted by Photius, about 870 A.D.): "Apollonius, who lived in the time of Philopator (221 to 205 B.C.), became most famous as an astronomer; he was called ϵ since the figure of ϵ is related to the figure of the moon which he investigated most accurately." Then Vettius Valens, who wrote about 160 A.D., says that he used the tables "of Hipparchus for the sun, of Sudines, Kidenas, and Apollonius for the moon, and also Apollonius for both types (of eclipses)." Finally the "Refutation of all Heresies" (written about 230 A.D.) quotes a figure for the distance from the surface of the earth to the moon proposed by Apollonius. The only substantial information about Apollonius' astronomical work comes from the *Almagest* [...].[2]

[2] O. Neugebauer, *The Equivalence of Eccentric and Epicyclic Motion According to Apollonius*, «Scripta Mathematica», XXIII, 1959, pp. 5-21, on p. 5; reprinted in Id., *Astronomy and History: Selected Essays*, New York-Berlin, Springer, 1983, pp. 335-351. See also Id., *Apollonius' Planetary Theory*, «Communications on Pure and Applied Mathematics», VIII, 1955, pp. 641-648; reprinted in Id., *Astronomy and History*, cit., pp. 311-318.

The passages that refer to the Moon occur in texts written hundreds of years after the lifetime of Apollonius, and only one of them can be considered of any significance. It appears in a work by Vettius Valens, an astrologer who wrote in Greek, but its usefulness in this context is undermined by Jones's argument that one should read "Apollinarius" instead of Apollonius. Now Sudines and Kidenas are mentioned in several Greek sources as experts on Babylonian astronomy, and Apollonius is mentioned in another passage in Vettius Valens: "For even Apollonius, who worked out [tables] in accordance with the phenomena using ancient observations and demonstrations of complicated periodic restitutions [?] and spheres, and who brought censure upon many, admits to erring by one degree or even two."[3] Jones argues that both passages in Vettius Valens refer to the same person and suggests that it is "highly probable" that Apollinarius was intended, rather than Apollonius. There are several texts that mention this Apollinarius (late 1st, or early 2nd, century A.D.): none of his works is extant, but some evidence for his contributions to astronomy is preserved in an early commentary on Ptolemy's *Almagest*.[4]

The principal claim for Apollonius's contribution to astronomy is the theorem on stationary points for planetary motion in Ptolemy's *Almagest* XII.1 (see Fig. 1). Neugebauer infers that the discussion of the equivalence of the eccentric and epicyclic lunar models in *Almagest*, IV.5 is due to Apollonius, acknowledging that he is not mentioned there. It would also seem to follow that the equivalence of the eccentric and epicyclic solar models was known to Apollonius, if not discovered by him.[5] This fits in with the "history" of planetary astronomy by Simplicius (6th century A.D.): Plato gave the astronomers the task of "saving the phenomena," followed by the homocentric models of Eudoxus and Calippus, which were undermined by Autolycus (*ca.* 300 B.C.) who claimed that Mars and Venus are seen to be bigger (to be understood as brighter) in the middle of their retrograde arcs.[6]

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[3] Translated in A. Jones, *Ptolemy's First Commentator* (Transactions of the American Philosophical Society, 80.7), Philadelphia, American Philosophical Society, 1990, p. 14.

[4] Id., *Ptolemy's First Commentator*, cit. in note 3, pp. 12-17. See also G.J. Toomer, *Galen on the Astronomers and Astrologers*, «Archive for History of Exact Sciences», XXXII, 1985, pp. 193-206, on pp. 199 and 203. For the other references to Apollinarius in ancient literature (none of which concerns planetary theory), see O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin-New York, Springer, 1975, p. 601 n. 2.

[5] J. Evans, *The History and Practice of Ancient Astronomy*, New York-Oxford, Oxford University Press, 1998, p. 212; O. Pedersen & M. Phil, *Early Physics & Astronomy*, New York, American Elsevier, 1974, pp. 81-84.

[6] T.L. Heath, *Aristarchus of Samos*, Oxford, Clarendon Press, 1913, p. 222. For detailed dis-

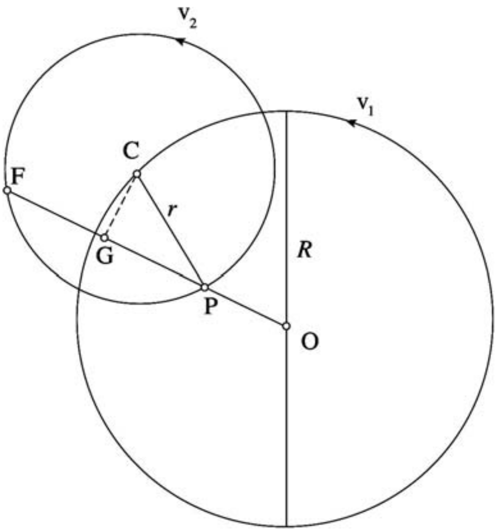

Fig. 1 – This figure illustrates Apollonius's theorem. The observer, O, is at the center of a concentric deferent whose radius is *R*; the radius of the epicycle is *r* and its center is C. The constant velocity on the deferent is v_1 and the constant velocity on the epicycle is v_2. According to Apollonius's theorem, the planet is at stationary point, P, if and only if

$$v_1 / v_2 = GP / PO,$$

where GP is half the chord which, extended, reaches O.

Then, according to the standard account, Apollonius set astronomy on a different course by showing the equivalence of the eccentric and epicyclic models, and parameters for these solar and lunar models were added by Hipparchus.[7] Ptolemy accepted Hipparchus's results for the Sun, but modified the lunar parameters. The methods used by Hipparchus were also used by Ptolemy, but Hipparchus failed to come up with satisfactory models for the five planets.

I have expressed scepticism about many aspects of the standard account and Simplicius's reliability as a source for historical data. In particular, what makes this story especially attractive is that, according to Simplicius, Autolycus presented observational data that rendered all homocentric models untenable, and then Apollonius introduced an entirely new way "to save the phenomena." But is Simplicius's claim on behalf of Autolycus credible? I think not. As a matter of fact, the brightness of Mars varies quite noticeably but that of Venus varies hardly at all (since the phases counteract the variation in distance), and the sizes of planets are not

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cussion of the text of Simplicius, see A.C. BOWEN, *Simplicius' Commentary on Aristotle*, De Caelo *2:10-12: An Annotated Translation (Part 2)*, «Sciamvs», IX, 2008, pp. 25-131, on pp. 72-73 and 104-105. Strictly speaking, Venus is not visible at the mid-point of its retrograde arc.

[7] See O. NEUGEBAUER, *The Equivalence of Eccentric and Epicyclic Motion*, cit. in note 2, p. 21: "Thus, it seems to me that all the evidence points to Apollonius as the founder of Greek mathematical astronomy which provided the starting point for all further progress in the understanding of our planetary system." The evidence for either the eccentric or the epicyclic model before Apollonius is very weak and based on sources of dubious reliability from late Antiquity: see, e.g., ID., *A History of Ancient Mathematical Astronomy*, cit. in note 4, pp. 694-697.

specified in any ancient text before Ptolemy's *Planetary Hypotheses*.[8] Moreover, there is no occurrence of a term in Greek or Latin for retrograde motion until much later (see notes 15 and 16, below). It seems, then, that Simplicius depended on later developments in astronomy and ascribed them to Autolycus. In Ptolemy's models Mars and Venus both have large epicycles which means that their distances from the Earth vary, and their ratios of greatest to least distance are approximately the same.

3. Methodological Considerations

At this point some methodological remarks are in order. First and foremost the success of Ptolemy's *Almagest* and the paucity of earlier textual evidence in Greek have tended to obscure Ptolemy's innovations for, in hindsight, his methods often seem to be the "natural" way to pursue astronomical matters.[9] Even the basic idea of presenting auxiliary tables (mean motions and anomalies) for computing planetary positions is not attested before Ptolemy. In my view one must resist giving credit to Ptolemy's predecessors for any aspect of the *Almagest* unless there are good reasons for doing so. The reliability of sources from late Antiquity, especially when they refer to astronomers who lived hundreds of years earlier, must be assessed before accepting them as historically valid. In particular, one needs strong corroborating evidence from sources contemporary with these early astronomers.

One might appeal to the fact that only a few texts from this early period have survived. Although it is sometimes said that the absence of evidence is not evidence of absence,[10] it is unreasonable to demand "evidence of absence." One can hardly expect an ancient scholar to report the absence of a concept that was introduced after his time. What about ancient texts

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[8] B.R. Goldstein, *Commentary on: 'The Status of Models in Ancient and Medieval Astronomy'*, «Centaurus», L, 2008, pp. 184-188; Id., *Saving the Phenomena: The Background to Ptolemy's Planetary Theory*, «Journal for the History of Astronomy», XXVIII, 1997, pp. 1-12; Id., *The Pre-Telescopic Treatment of the Phases and Apparent Size of Venus*, «Journal for the History of Astronomy», XXVII, 1996, pp. 1-12; Id., *The Arabic Version of Ptolemy's Planetary Hypotheses* (Transactions of the American Philosophical Society, 57.4), Philadelphia, American Philosophical Society, 1967. See also note 26, below, and A.C. Bowen, *Simplicius and the Early History of Greek Planetary Theory*, «Perspectives on Science», X, 2002, pp. 155-167.

[9] See B.R. Goldstein, *What's New in Ptolemy's* Almagest?, «Nuncius», XXII, 2007, pp. 261-285.

[10] Cf. C. Sagan, *The Demon-haunted World: Science as a Candle in the Dark*, New York, Ballantine Books, 1995, 2nd ed. 1997, pp. 212-213.

that may yet be discovered? It is notoriously difficult to "predict" what might be in such texts. Rather than relying on expectation, I seek to determine the earliest date for which there is positive evidence of some concept; to be sure, the concept may have been known earlier but, until specific evidence is identified, one can only speculate. History should be set on a more solid foundation than speculation (which often builds on itself).

In light of this methodology, it is important to take into account the fact that there is no allusion to planetary retrogradation in a Greek text composed prior to the 1st century B.C., let alone to a theorem by Apollonius on this matter.[11] But, under the assumption that Apollonius had a theorem on stationary points, one would expect some echo of it in the period after him. If there is no such echo of a discovery, it is hard to claim that it is historically significant. Moreover, Ptolemy's attribution of this theorem to Apollonius is peculiar: it was not Ptolemy's custom to ascribe theorems to a named predecessor, and this is the only exception in the *Almagest*.[12] On the other hand, Ptolemy names observers (for the reliability of their data is an issue), and mentions Hipparchus on a number of occasions, often to criticize his theoretical work.[13]

What about the role of Apollonius of Perga? On the positive side, the text of *Almagest* XII.1 has no significant variants either in the Greek original or in an Arabic translation of the 9th century.[14] So Ptolemy's authority would seem to support the role of Apollonius as the discoverer of the theorem on stationary points. But does this attribution cohere with other evidence? First, the association of Apollonius with astronomy (other

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[11] It may well be the case that knowledge of retrogradation and stationary points reached astronomers in the Greco-Roman world in the 1st century B.C. when Babylonian astronomical sources became available to them, rather than from direct observation of planetary phenomena. On evidence for Babylonian astronomical methods in Greek texts see, e.g., A. JONES, *Astronomical Papyri from Oxyrhynchus*, Philadelphia, American Philosophical Society, 1999.

[12] For example, Menelaus (*ca.* 100 A.D.) is not credited for his theorem on spherical triangles in *Almagest* I.13 even though he is mentioned as an observer in *Almagest* VII.3. See O. NEUGEBAUER, *A History of Ancient Mathematical Astronomy*, cit. in note 4, p. 27.

[13] B.R. GOLDSTEIN & A.C. BOWEN, *The Role of Observations in Ptolemy's Lunar Theory*, in *Ancient Astronomy and Celestial Divination*, ed. by N.M. Swerdlow, Cambridge, Mass., MIT Press, 1999, pp. 341-356. In his *Geography* Ptolemy ascribes a particular projection to Marinus (which is a theoretical matter) in addition to citing Hipparchus for measurements of geographical latitude: see J.L. BERGGREN & A. JONES (trs.), *Ptolemy's Geography*, Princeton-Oxford, Princeton University Press, 2000. I am most grateful to Len Berggren for bringing this reference to my attention.

[14] J.L. HEIBERG (ed.), *Claudii Ptolemaei Syntaxis mathematica*, Leipzig, Teubner, 1898-1903, vol. II, p. 450; for the occurrences of Apollonius in the Arabic version of the *Almagest* by al-Ḥajjāj (*ca.* 830 A.D.), see Leiden, MS Or. 680, ff. 182b:31 and 183b:23.

than this passage) is extremely weak. Moreover, in the Greco-Roman world the earliest occurrence of a term for a planet's station is in a text of Diodorus Siculus (1^{st} century B.C.),[15] and the earliest for retrograde motion is in Cicero (1^{st} century B.C.).[16] It is, of course, true that the Babylonian astronomers were well aware of planetary phenomena including the stations long before these first occurrences, but that does not mean that the Greeks considered these phenomena in a scientific context.

4. Eccentrics and Epicycles

If we set aside Apollonius's theorem, is there independent evidence for the equivalence of the eccentric and epicyclic hypotheses? Generally, it is claimed that this equivalence was known to Hipparchus (based on evidence in the *Almagest*), but a careful reading of the relevant passages does not support this view. The key text comes in *Almagest* IV.11, where the subject is the determination of the lunar parameters based on observations of triples of eclipses. Ptolemy asserts that Hipparchus found one ratio for the radius of the lunar eccentric circle to its eccentricity, namely, 3144 to $327\,^2/_3$ ($\approx$ 60 to 6;15), but a different ratio for the radius of the lunar deferent to the lunar epicyclic radius, namely, $3122\,^1/_2$ to $247\,^1/_2$ ($\approx$ 60 to 4;46).

Ptolemy comments:

> Such a discrepancy cannot, as some think, be due to some inconsistency between the [epicyclic and eccentric] hypotheses [...].[17]

Ptolemy does not say that Hipparchus was aware of the equivalence of the two models; rather, he says that he, Ptolemy, knows they are equivalent and so the discrepancy must be due to something else. Moreover, Ptolemy does not reproduce the method that Hipparchus used

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[15] Diodorus, *Bibliotheca historica*, I.81: *stêrigmous*; C.H. Oldfather (ed. and tr.), *Diodorus of Sicily*, Cambridge, Mass., Harvard University Press, 1933, vol. I, pp. 278-279. The data base, *Thesaurus Linguae Graecae*, was searched for this term.

[16] Cicero, *De natura deorum*, II.51: *progressus et regressus*; H. Rackham (ed. and tr.), *Cicero: De natura deorum*, Cambridge, Mass., Harvard University Press, 1933, pp. 172-173.

[17] *Almagest*, IV.11; G.J. Toomer (tr.), *Ptolemy's Almagest*, New York-Berlin, Springer, 1984, p. 211. In standard sexagesimal notation a semicolon is used to separate the integer from the fractional part, e.g., 6;15 is equivalent to $6 + {}^{15}/_{60}$. Cf. A. Aaboe, *Episodes from the Early History of Mathematics*, Washington, DC, The Mathematics Association of America, 1964, p. 16.

to derive these parameters from the stated observations; instead, Ptolemy shows that with his own tools the discrepancy disappears, suggesting that the discordant data cited in the name of Hipparchus is due to some error (or errors) in Hipparchus's derivation. But it does not seem that Ptolemy had access to the details of Hipparchus's derivation.

Neugebauer comments on this passage:

> It is extremely unlikely that Hipparchus intended to disprove the equivalence theorem of Apollonius through numerical examples taken from two different triples of eclipses. The only plausible motive for Hipparchus' investigation seems to be the question whether the eccentricity of the lunar orbit remains constant or not [...]. Hipparchus' results (obtained from either of the two equivalent models) must have led him to the conclusion that the lunar epicycle shows a variable diameter.[18]

Since Neugebauer accepts the role of Apollonius, he is then left to conjecture the motives of Hipparchus, assuming Hipparchus to be aware of the theorem proved by his predecessor. But the need for this conjecture vanishes if the assumption that Apollonius had proved this equivalence is false.

In *Almagest* III.4 Hipparchus is associated with an eccentric model for solar motion; Ptolemy derives the parameters for this model from data for the length of the seasons that he ascribes to Hipparchus, and these data are found in the astronomical literature independent of the *Almagest*.[19] Ptolemy begins this passage by saying that "these problems [finding the solar eccentricity and apogee] have been solved by Hipparchus with great care," and ends by saying that "the above conclusions [presumably that the solar eccentricity is $^1/_{24}$ of the radius of the eccentric circle, and that the apogee is at Gem. 5;30°] are in agreement with what Hipparchus says."[20] But Ptolemy does not claim that Hipparchus came to these results

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18 O. NEUGEBAUER, *The Equivalence of Eccentric and Epicyclic Motion*, cit. in note 2, pp. 18-19.

19 See GEMINUS, *Introduction to the Phenomena*, I.13-17; J. EVANS & J.L. BERGGREN (trs.), *Geminos's Introduction to the Phenomena*, Princeton-Oxford, Princeton University Press, 2006, pp. 116-117; G. AUJAC (ed. and tr.), *Géminos: Introduction aux phénomènes*, Paris, Les Belles Lettres, 1975, pp. 4-5; J. DUPUIS (ed. and tr.), *Théon de Smyrne: Exposition des connaissances mathématiques utiles pour la lecture de Platon*, Paris, Hachette, 1892, p. 249. On the key datum of $94^1/_2$ days from vernal equinox to summer solstice as coming from a Babylonian scheme, see A.C. BOWEN & B.R. GOLDSTEIN, *Meton of Athens and Astronomy in the Late Fifth Century B.C.*, in *A Scientific Humanist: Studies in Memory of Abraham Sach*, ed. by E. Leichty, M. de J. Ellis, & P. Gerardi, Philadelphia, University Museum, 1988, pp. 39-81, on p. 69.

20 G.J. TOOMER, *Ptolemy's Almagest*, cit. in note 17, p. 156.

in the same way that Ptolemy did. Moreover, Ptolemy does not indicate if Hipparchus considered an epicyclic model for the Sun. In *Almagest* IV.5 where, for the Moon at syzygy (i.e., conjunction with, or opposition to, the Sun), the equivalence of the eccentric and epicycle models is stated, Hipparchus is mentioned once:

> In this first part of our demonstrations we shall use the methods of establishing the theorem which Hipparchus, as we see, used before us. We too, using three lunar eclipses, shall derive the maximum difference from mean motion and the epoch of the [moon's position] at the apogee, on the assumption that only this [first] anomaly is taken into account, and that it is produced by the epicyclic hypothesis.[21]

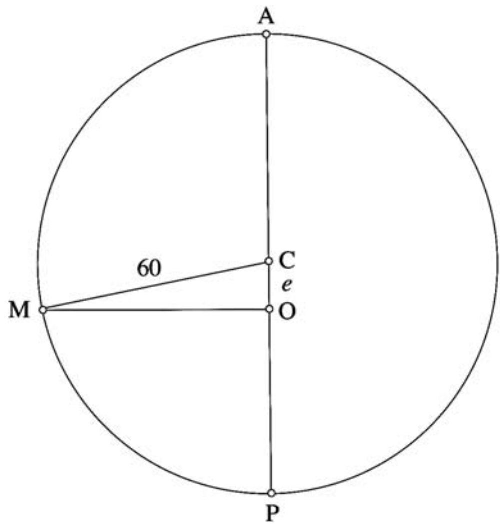

Fig. 2 – This figure illustrates an eccentric model where C is the center of the eccentric circle whose radius is 60, A is the apogee, O is the observer, and CO is the eccentricity, *e*. The mean motion, counted from the apogee, is ∠ACM and the true motion is ∠AOM: the difference (or equation) is ∠CMO. The maximum equation, ∠CMO, takes place when OM is perpendicular to AO. In modern terms,

$$\sin \alpha = e/60, \quad [1]$$

where α = ∠CMO. The parameters that may be used to characterize the eccentric model are the value for the eccentricity, *e*, and the value for the maximum equation, α; from eq. [1] it follows that each of them determines the other. A similar argument holds for the epicyclic model.

This passage certainly suggests that Hipparchus used the same method as did Ptolemy, but it is not supported by the text of *Almagest* IV.11. Ptolemy's appeal in *Almagest* IV.5 to the "maximum difference from mean motion," may simply be due to Ptolemy's understanding that the eccentricity and the maximum equation of anomaly are two ways to characterize the same eccentric model (see Fig. 2).[22] This claim on behalf of Hipparchus seems to be an inference by Ptolemy

[21] *Ibid.*, p. 181.

[22] It is quite common for scientists (and others) to make seemingly innocent anachronistic adjustments, as when one translates "half" in an ancient text as "0.5," long before decimal fractions were invented.

based on the fact that both he and Hipparchus started with observations of triples of eclipses.

The problem with the assertion that Hipparchus used the same method as Ptolemy is that Ptolemy's method depends on the concept of mean position for the Moon, but there is no evidence for Hipparchus's awareness of this concept in any passage in the *Almagest*, and some that argue against it. The data for each lunar eclipse used by Hipparchus are only the true lunar position at eclipse-middle, and the time of its occurrence.[23] Nothing about a mean lunar position is ascribed to Hipparchus. Moreover, in a passage concerning the length of the solar year in *Almagest* III.1, Ptolemy describes Hipparchus's method for finding the position of the Moon at eclipse-middle:

> For he [Hipparchus] uses lunar eclipses which were observed to take place near [specific] fixed stars to compare the distance of the star called Spica in advance of the autumnal equinox at each [eclipse]. By this means he thinks he finds, on one occasion, a distance of $6^1/_2$°, the maximum in his time, and on another a distance of $5^1/_4$°, the minimum [in his time] [...]. To take a single example: from the eclipse observation in the thirty-second year of the Third Kallippic Cycle [–145 Mar 24] which he adduces, he claims to find that Spica is $6^1/_2$° in advance of the autumnal equinox, whereas from the eclipse observation in the forty-third year of that cycle [–134 Mar 23/24] he claims to find it is $5^1/_4$° in advance. Likewise, in order to carry out the computations for the above, he adduces the spring equinoxes which he had accurately observed in those years. This was in order that from the latter he could find the position of the sun at the middle of each eclipse, from these the positions of the moon, and from the positions of the moon those of the stars.[24]

Note that Hipparchus's procedure for finding the position of the Moon at eclipse-middle, according to Ptolemy, does not appeal to a mean position of the Sun or the Moon.[25]

Extant Greek and Latin astronomical texts from the period between Apollonius and Ptolemy, few though they may be, do not discuss the equivalence of these two models. For example, Pliny (1st century A.D.)

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[23] *Almagest* IV.11; G.J. Toomer, *Ptolemy's Almagest*, cit. in note 17, pp. 211-216. It is likely that the positions were computed from the observed times.

[24] *Ibid.*, p. 135.

[25] For various ways to compute a true solar position without appealing to a mean solar position, see B.R. Goldstein, *What's New in Ptolemy's Almagest?*, cit. in note 9, pp. 278-281. See also A. Jones, *Hipparchus's Computations of Solar Longitudes*, «Journal for the History of Astronomy», XXXII, 1991, pp. 101-125.

describes an epicyclic model for planetary motion. Pliny's text is rather confused in many ways: there is no hint of the equivalence of epicyclic and eccentric models, but he does claim that planets vary in apparent size, appearing larger as they come nearer to the Earth.[26]

Another discussion of the equivalence of the two models comes in a text by Theon of Smyrna (2nd century A.D.) where the following is added:

> In this way Adrastus shows that the phenomena are explained by the two hypotheses, that of the eccentric circle and that of the epicycle. Hipparchus made the remark that the reason that the same phenomena follow from such different hypotheses, that of the eccentric circles and that of the concentric circles and epicycles, is worthy of the attention of the mathematician. Adrastus has shown that the hypothesis of the eccentric circle is a consequence of that of the epicycle; but I say further that, the hypothesis of the epicycle is also a consequence of that of the eccentric circle.[27]

It is customary to discard this report on the grounds that Theon of Smyrna is simply misinformed since "we know" that this equivalence had already been proved by Apollonius.

5. Concluding Remarks

Finally, let us consider the two passages in *Almagest* XII.1 where the name Apollonius appears. Here is the first passage in Toomer's translation:

> Now that we have demonstrated the above, the appropriate sequel would be to examine the greatest and least retrogradations associated with each of the 5 planets, and to show that the sizes of these, [as computed] from the above hypotheses [i.e., models], are in as close agreement as possible with those found from observations.

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[26] Pliny, *Naturalis historia*, II.64 and 68-73; H. Rackham (ed. and tr.), *Pliny: Natural History*, Cambridge, Mass., Harvard University Press, 1938, vol. I, pp. 212-219. Cf. O. Neugebauer, *A History of Ancient Mathematical Astronomy*, cit. in note 4, pp. 802-805. Although the variation in planetary sizes as a function of their distances from the Earth is easy to calculate from the data in Ptolemy's *Planetary Hypotheses*, it was not done until the Middle Ages: see B.R. Goldstein & N.M. Swerdlow, *Planetary Distances and Sizes in an Anonymous Arabic Treatise Preserved in Bodleian Ms. Marsh 621*, «Centaurus», XV, 1970-71, pp. 135-170; reprinted in B.R. Goldstein, *Theory and Observation in Ancient and Medieval Astronomy*, London, Variorum, 1985, essay VI. See also Id., *Levi ben Gerson and the Brightness of Mars*, «Journal for the History of Astronomy», XXVII, 1996, pp. 297-300.

[27] R. Lawlor & D. Lawlor (trs.), *Mathematics useful for understanding Plato by Theon of Smyrna*, San Diego, Wizards Bookshelf, 1979, pp. 107-108; cf. J. Dupuis, *Théon de Smyrne: Exposition des connaissances mathématiques utiles pour la lecture de Platon*, cit. in note 19, p. 269. O. Neugebauer (*A History of Ancient Mathematical Astronomy*, cit. in note 4, p. 264 n. 3) dates Adrastus *ca.* 100 A.D.

In the definition of this kind of problem, there is a preliminary lemma demonstrated (for a single anomaly, that related to the sun) by a number of mathematicians, notably Apollonius of Perge [καὶ οἵ τε ἄλλοι μαθηματικοὶ καὶ Ἀπολλώνιος ὁ Περγαῖος], to the following effect [...].[28]

A more literal translation of the key phrase would be "both other mathematicians and Apollonius of Perga,"[29] A parallel usage of this phrase occurs in *Almagest* IX.1 in the context of previous efforts to construct a planetary theory:

Although he [Hipparchus] investigated the theories of the sun and the moon [...], he did not make even a beginning in establishing theories for the five planets, not at least in the writings that have come down to us. All that he [Hipparchus] did was to make a compilation of the planetary observations arranged in a more useful way, and to show by means of these that the phenomena were not in agreement with the hypotheses of the astronomers of that time. For, we may presume, he thought that one must not only show that each planet has a twofold anomaly, or that each planet has retrograde arcs which are not constant, and are of such sizes (whereas other astronomers [ἄλλων μαθηματικῶν] had constructed their geometrical proofs on the basis of a single unvarying anomaly and retrograde arc) [...].[30]

This passage is unique in offering potential support for the claim that Hipparchus was aware of retrograde motion. Ptolemy tells us that, as far as he was aware, Hipparchus was not successful in constructing a planetary theory, and that he only compiled a list of planetary observations and criticized theories that were available in his time, i.e., those by the "other mathematicians." But this group of "other mathematicians" is not identified, and it is possible that Ptolemy recast what they did in his own terms. There is a connection between this passage in *Almagest* IX.1 and the passage in XII.1, for the expression "a single unvarying anomaly" appears

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[28] G.J. Toomer, *Ptolemy's Almagest*, cit. in note 17, p. 555. For modern discussions of Apollonius's theorem, see O. Neugebauer, *Apollonius' Planetary Theory*, cit. in note 2; A. Aaboe, *On Babylonian Planetary Theories*, «Centaurus», V, 1958, pp. 209-277, on pp. 274-275. O. Neugebauer (*A History of Ancient Mathematical Astronomy*, cit. in note 4, p. 264) suggested that there is a link between Apollonius's work on conic sections and the proof of the theorem in *Almagest* XII.1, but G.J. Toomer (*Ptolemy's Almagest*, cit. in note 17, p. 556 n. 3) rejects this suggestion because Ptolemy offers his own proof, in opposition to that of his predecessors.

[29] For a detailed and insightful discussion of this passage, see A.C. Bowen, *La scienza del cielo nel periodo pretolomaico*, in *Storia della scienza*, 10 vols., vol. I: *La scienza antica*, Rome, Istituto della Enciclopedia Italiana, 2001, pp. 806-839, on pp. 821-822.

[30] G.J. Toomer, *Ptolemy's Almagest*, cit. in note 17, p. 421.

in both of them, but there is no good reason to suppose that the "other mathematicians" in the two passages refer to the same group. The expression, "retrograde arc," with respect to Hipparchus is an inference by Ptolemy, unsupported by any reference to a work by Hipparchus. The general assumption is that Ptolemy was well informed about the works by previous astronomers but, as this passage indicates, this may only be true to a limited extent. Indeed, it is entirely possible that Ptolemy depended, at least in part, on some intermediary (whose reliability cannot be determined) for his knowledge of previous work in astronomy. Be that as it may, this passage provides evidence that Ptolemy assumed Hipparchus knew about retrograde motion, and this is consistent with the claim in *Almagest* XII.1 that the theorem on stationary points is due to Apollonius. But, even if Hipparchus were aware of retrograde motion, it hardly follows that he knew Apollonius's theorem. At best, we learn what Ptolemy believed, but it does not establish the fact. Moreover, one should bear in mind that Ptolemy was an astronomer, not a historian of astronomy.

Here is the second mention of Apollonius in *Almagest* XII.1:

> For this purpose Apollonius proposes the following lemma [...].[31]

This lemma has to do with an inequality in a triangle (a ratio of lengths is greater than a ratio of angles), and Ptolemy includes a proof that he ascribes to Apollonius: it is a matter of pure mathematics, and its original context need not have been astronomical.

In conclusion, we are left with a few possibilities for interpreting the passage at issue in *Almagest* XII.1:

1. Ptolemy is simply misinformed, and the theorem is due to a later astronomer.
2. Ptolemy intended to say Apollinarius, but at an early stage in the transmission of the text, Apollonius was substituted for Apollinarius. This has the difficulty that, as far as I can determine, Apollinarius is not associated with planetary theory.
3. The name of Apollonius was interpolated by a copyist (perhaps a marginal gloss that was later incorporated into the text). The two occurrences would not have to be the work of a single copyist.[32]

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[31] G.J. TOOMER, *Ptolemy's Almagest*, cit. in note 17, p. 558.

[32] *Ibid.*, pp. 4-5: "During the course of making the translation, I became convinced that the text contains a number of interpolations, which must go back to antiquity, since they are in the whole

4. Ptolemy is well informed even though no trace of this theorem is to be found in the Greco-Roman world prior to Ptolemy.

I find the fourth possibility least likely, but others may wish to defend it by appealing to arguments I have not considered. In any event, it seems that the role of Apollonius in the history of astronomy has been exaggerated. Moreover, the evidence for Hipparchus's knowledge of the equivalence of the eccentric and epicyclic models is very weak, and there is no evidence for his familiarity with the theorem on stationary points. In sum, the history of Greek planetary astronomy needs to be rewritten.

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manuscript tradition, both Greek and Arabic." Toomer does not discuss whether this applies to *Almagest* XII.1.