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HISTORICAL PERSPECTIVES ON COPERNICUS'S ACCOUNT OF PRECESSION

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It has seemed strange to some modern historians of science that the first response to Copernicus's *De revolutionibus* had as much to do with his account of the motion of the starry sphere (including precession) as with his heliocentric hypothesis. Indeed, in Rheticus's *Narratio prima* (1540), the earliest published report of Copernican astronomy, the first two topics to be considered concern the motion of the fixed stars and the tropical year.[1] But in the context of the astronomical tradition at the time, this response was quite sensible, for the motion of the starry sphere was a recognized problem in the astronomical community whereas geocentrism was not.[2] For us, however, Copernicus's treatment of precession addresses a non-existent phenomenon, namely, the trepidation of the equinoxes or a variable rate of precession, and so it has been hard to see this as a significant scientific contribution. But astronomers at his time, including Copernicus himself, were responding primarily to a literary tradition of scientific treatises, rather than to nature as revealed by their own astronomical observations.[3] As Kepler put it in the *Astronomia nova* (1609), chap. 14: "Copernicus, ignorant of his own riches, ever took it upon himself to express Ptolemy, not the nature of things, to which, nonetheless, he of all men came closest." Copernicus was hardly alone in this attitude towards nature, for astronomical observations were often made merely to confirm a theory rather than to test it; in other cases, no new observations were sought because a new theory was 'tested' against an earlier theory which was deemed successful in accounting for the observed phenomena but was inadequate in some other respect (e.g., it violated the principle of uniform circular motion).

The principal ancient source for the discussion of precession is the *Almagest*: Ptolemy defined precession as the uniform increase in the longitudes of the fixed stars, and derived a value for it of $1^\circ/100$ years (*Almagest*, vii.2). Moreover, Ptolemy argued that the proper measure of time is the tropical year, and that the length of the tropical year is $365;14,48^d$ (*Almagest*, iii.1). He explicitly denied the validity of the sidereal year and did not calculate a value for its length. In fact, precession is closely related to the difference between the sidereal year and the tropical year and, from a modern point of view, it is entirely arbitrary to prefer one over the other.[4] But Ptolemy's treatment of precession is based on the tropical year, and his value for the tropical year is not very good. Curiously, the value for the sidereal year that can be derived from his values for the tropical year and precession, $365;15,24,32^d$,[5] is better, and this is what caused the problem.

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When new observations were made by Muslim astronomers in the ninth century, it was found that the value for the sidereal year was close to what it would have been for Ptolemy, but the value for the tropical year seemed to have changed from his time to theirs. One solution was to harmonize the data, i.e., to claim that the tropical year had changed since the time of Ptolemy while the sidereal year had remained constant.[6] This solution entailed a variation in precession over time, and this variable precession is now called trepidation. In this tradition, the most successful model that accounted for the data is Copernicus's, and it succeeded where the competitors available in his day had failed (relatively speaking). However, it is important to note that not all medieval astronomers accepted a theory of variable precession: some preferred to keep to the tropical year and a fixed rate of precession as Ptolemy had (but with new values for both parameters); others preferred to consider the sidereal year as their measure of time. For example, Levi ben Gerson (southern France, d. 1344) rejected Ptolemy's value for precession even for the epoch of Ptolemy, and added: "We have shown that this motion is uniform and there is no need to introduce a variable motion for the stars."[7]

It would seem that the first treatise to address the difficulty in Ptolemy's values for precession and the tropical year is *On the solar year*, a text ascribed in Latin and in some Arabic sources to Thābit Ibn Qurra (d. 901) but, with the recovery of the Arabic text, it has been argued persuasively that this treatise was due to Thābit's patrons, the Banū Mūsā.[8] In this treatise there is no trepidation; the sidereal year is preferred over the tropical year;[9] a value is stated for the sidereal year; and a new value is found for precession that differed from that of Ptolemy. Instead of Ptolemy's value for the tropical year of 365;14,48^d, the author of this treatise preferred a sidereal year of 365;15,23,34,33^d.[10] The value for precession in this treatise is given as 0;0,49,39°/year: it is derived by dividing 13;12° by 957 years (of 365 days), and rounding the result to two significant sexagesimal places. While the result of this division is the same in both the Arabic and the Latin versions, the Latin gives the numerator as 13;23° and the denominator as 958 years — the Arabic version has the better readings.[11] The parameter, 1°/72;30 years, does not appear in this text, but it is of interest to note that 13;12°/957 years is exactly the same as 1°/72;30 years (13;12 × 72;30 = 957). With this parameter for precession and the value for the length of the sidereal year, a 'mean' value for the tropical year of 365;14,33,12^d is then computed.[12]

A second treatise in Latin was attributed to Thābit, *On the motion of the eighth sphere*, but no Arabic version has yet been discovered, and the attribution to Thābit has been seriously questioned.[13] In this treatise the sidereal year is fixed, and there is a variable precession.[14] The tables for computing the variable precession according to this model also appear in the Toledan tables that were widely used in medieval Europe.[15]

In Spain, Ibn al-Kammād (12th century) had a theory of variable precession that bears some resemblance to that of *On the motion of the eighth sphere*. According to Ibn al-Kammād, the sidereal year is fixed and precession varies in a roughly sinusoidal way with a maximum of 9;59°. Ibn al-Kammād's treatise and tables are not extant in Arabic, but they do survive in a Latin manuscript.[16]

A key Spanish text for the history of theories of variable precession is chapter 49 of the Castilian canons to the tables produced for Alfonso X of Castile (reigned: 1252–84), which is devoted to a description of a table for this phenomenon. But, since the canons do not provide parameters and the corresponding tables do not survive, it is not easy to decide precisely what was intended. Still, the text seems to say that the sidereal year is fixed and that there was a variable term for precession.[17] On the basis of this meager evidence, it cannot be decided whether the table was that of Ibn al-Kammād or that of *On the motion of the eighth sphere*. This does not exclude other possibilities, but they seem less likely.

The Latin version of the Alfonsine tables, produced in Paris in the 1320s, became the basic set of tables for European astronomers from the fourteenth to the sixteenth centuries.[18] There has been considerable controversy concerning the relationship of these tables to texts or tables that were produced by astronomers working for Alfonso X; indeed, it has even been claimed that the Alfonsine tables as we have them in Latin were entirely the product of Parisian astronomers and that they had no significant debt to their Castilian predecessors.[19] The Parisian Alfonsine tables introduced two terms for precession, a linear term and an oscillatory term. The linear term has a period of 49,000 years (of $365;15^d$) which may also be expressed as 1°/136;6,40 years: this corresponds to the difference between a tropical year of about $365;14,33,10^d$ and a Julian year of $365;15^d$.[20] While in itself this has no astronomical significance since the length of the calendar year is merely conventional, when added to the oscillatory term as evaluated for the time of Alfonso, the value for precession according to the Alfonsine tables, 17;8°, yields good results for star positions when added to those of the star catalogue in Ptolemy's *Almagest*.[21] The oscillatory term has a period of 7000 years and a maximum of 9°.[22] In this model the tropical year is fixed and there is a variable precession.

A crucial text for the Parisian Alfonsine group was written by John of Murs in 1321, and it is entitled *Expositio intentionis regis Alfonsii circa tabulas ejus*.[23] In this treatise, as in the Parisian Alfonsine tables, the tropical year is taken as the measure of time which seems to be different from the usage in the Castilian canons. In one section of this text that has resisted interpretation, John listed values for precession according to his predecessors:

> Thus, he [Alfonso] specified their motion in relation to the eighth sphere and, with respect to this [sphere], the motion is continuous (*continue*), uniform (*equaliter*), [and advances] by 0;0,0,4,20,41,17,12,27° every day in the order of the signs, which is the diurnal motion Alfonso determined for the Sun. But, in fact, Ptolemy fixed it as one degree in 100 years, and al-Battānī [as one degree] in 70 years.[24] Thābit, who was farthest from the

> truth, posited by means of a motion of progress and regress (*accessus et recessus*) that it was no greater than 16;45°, which is wrong by one degree already in our time and, further in the future, there will be a greater error. Alfonso later posited that the motion was continuous [at the rate of] one degree in $72\frac{1}{2}$ years, which is considered more correct, and this is in agreement with those of recent times (*modernis*) who unwittingly follow Alfonso in saying this.[25]

There are many difficulties with this text, but the principal difficulties are the parameter 1°/72;30 years for precession and the ascription to Alfonso. Poulle says of this parameter "évaluation inexplicable",[26] and North considers it likely that the $72\frac{1}{2}$ is John of Murs's own parameter.[27] There is another difficulty concerning the expression "the motion is continuous, uniform" used by John of Murs in one place, and "the motion was continuous" in another: the significance of this distinction, if any, escapes me. It is possible that John of Murs intended the second expression to refer to an 'average' value, but this may be reading more into the text than is really there. Moreover, the "recent" scholars who also accepted this parameter have not been identified. As for precession at the rate of 1°/72;30 years: this is precisely the figure that can be derived from the treatise *On the solar year*. Is this a chance coincidence? I do not think so, but let us consider some other numbers mentioned by John of Murs.

In the paragraph quoted above we find 0;0,0,4,20,41,17,12,27°; this number can be derived, very nearly, as follows:

$$360°/(49{,}000 \times 365;15) = 0;0,0,4,20,41,17,8,26°/\text{d},$$

and so it is the linear term for precession. The length of the tropical year is not stated explicitly in the Parisian Alfonsine tables, but John of Murs gives it as $365;14,33,9,59,20,7,30^{d}$,[28] or about $365;14,33,10^{d}$, which is very close to the length of the tropical year in *On the solar year*: $365;14,33,12^{d}$. It is clear that there are too many sexagesimal places in John of Murs's figure for this to represent any set of observations over the historical period; hence, as North suggested,[29] it is likely that it was extracted from a numerical table by carrying out the division to more than the appropriate number of significant places. Indeed, at the beginning of his *Expositio* John of Murs stated explicitly that he had seen the "Alfonsine" tables, but not the canons. Now the difference between John of Murs's figure for the tropical year and that found in *On the solar year* only accumulates to one day in 108,000 years: did John of Murs extract this parameter for the length of the tropical year from a numerical table that was in fact based on a very similar value? Alternatively, did John of Murs alter a figure slightly that he had found in an earlier text so that he would get one revolution in exactly 49,000 years for his linear term? The latter seems more likely, but it presupposes that John of Murs had a source in which this parameter appeared.

If we look again at the paragraph quoted from John of Murs, we notice that, with the exception of the first sexagesimal figure given to many places, the other figures are exact and are given in integers, or an integer together with a half. These figures seem to derive from a text, or texts, rather than having been extracted from a table or computed in some other way. Indeed, the values John of Murs attributed to Ptolemy[30] and to al-Battānī[31] can be confirmed in their treatises.[32] It then seems likely that the figure 1°/72;30 years also derives from a text — but there does not seem to be an earlier text in which this parameter is ascribed to Alfonso, as John of Murs would have us believe. Was John of Murs mistaken, or did his source mislead him? What possibly could be gained by attributing this parameter to Alfonso rather than to Thābit? John of Murs in this very paragraph alludes to *On the motion of the eighth sphere* which was attributed to Thābit, so there does not seem to be any particular reluctance to name him.

In John of Murs's text there is another value for precession: 1;22,40,53°/100 years which, according to him, is equivalent to 0;0,49,28°/year (accurately, 0;0,49,36,32°/year), a variant of 0;0,49,39°/year or 1°/72;30 years. This number seems to have been computed,[33] rather than copied from an earlier text: the difference between the sidereal year that is implicit the Toledan tables, 365;15,23,29^d (derived from the daily solar motion of 0;59,8,11,29,27°),[34] and John of Murs's value for the tropical year, 365;14,33,10^d, is 0;0,50,19^d and, at the rate of 0;59,8°/d, this corresponds to about 0;0,49,36°/year or 1°/72;34,50 years.[35]

While precise agreement is not always reached, the general sense of John of Murs's procedure does emerge. He had several sources, including one that depended on *On the solar year*, which he then tried to harmonize as best he could. In this case, the values for precession ascribed to his various predecessors are supposed to follow from a new variable precession model, but the computations are not explicitly carried out.

Finally, let us consider the Copernican model for precession in algebraic terms, ignoring the geometrical interpretation he gave it, for this suffices for purposes of calculation.[36] It is important to note that Copernicus fixed the sidereal year, and let the tropical year vary. The following formula for precession is taken from Mercier:[37]

$$\lambda = 5;32 + 360n/25816 - 1;10 \sin((360n/1717) + 13;30)$$

where n is the time measured in years of 365 days since the epoch, 31 Dec. 0 A.D. The linear term has a period of 25816 years (of 365 days) and the oscillatory term has a period of 1717 such years. According to Copernicus, the mean rate of precession is 0;0,50,12,5°/year (= 360°/25816 years), or very nearly 1°/72 years and, from a modern point of view, this is a rather good value for precession. But it was not unprecedented, for the parameter of precisely 1°/72 years appeared in the Tables of Barcelona composed in the fourteenth century[38] and, in a Viennese manuscript of the late fourteenth and early fifteenth centuries, a value close to 1°/72 years was computed on the basis of comparing data in the *Almagest* with data for 1306.[39] In other words, Copernicus analysed the reports available to him, including his own observations, and came up with a formula to account for these data by positing a variable precession that oscillated about the mean value for precession. Recently, Swerdlow and Neugebauer compared Copernicus's model with the data at his disposal, and showed that excellent agreement was achieved.[40]

By the early sixteenth century defects in the Alfonsine model for precession had been noted,[41] but this was not emphasized either by Copernicus or by Rheticus[42] (in contrast to John of Murs's criticism of Thābit's model, cited above). However, the main advantage of Copernicus over the Parisian Alfonsine group is that he returned to the original problem and solved it satisfactorily. The original problem was that Ptolemy's value for the tropical year was not good, but there was a reluctance to reject it outright because of his authority as an ancient and the impossibility of redoing his observations. The goal was to harmonize divergent values for precession from different epochs; there was little dispute on the length of the sidereal year, and so it was a good choice for the measure of time. But the Parisians fixed the tropical year and varied the sidereal year, and this was not the solution to any problem. Rather, it was based on a misunderstanding of the reasons for introducing variable precession. Since they wished to use the tropical year as their measure, they should also have accepted fixed values for precession and the sidereal year. Copernicus returned to the sidereal year and varied the tropical year with his oscillatory term for precession; in this way he came up with the best account of the data of which he and his followers were justly proud.

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2. See Regiomontanus's letter to Bianchini in N. Swerdlow, "Regiomontanus on the critical problems of astronomy", in *Nature, experiment, and the sciences*, ed. by T. H. Levere and W. R. Shea (Dordrecht, 1990), 165–95 (espec. pp. 170ff, 175ff).

3. For an example of the subordination of past observations to "tradition", see Copernicus, *Letter against Werner*: "However, by reason of the extreme slowness of this motion, the ancient mathematicians were unable to pass on to us a complete account of [the motion of the eighth sphere]. But if we desire to examine it, we must follow in their footsteps and hold fast to their observations, bequeathed to us like an inheritance. And if anyone on the contrary thinks that the ancients are untrustworthy in this regard, surely the gates of this art [astronomy] are closed to him" (transl. Rosen, *op. cit.* (ref. 1), 99). I intend to elaborate on this point in a future publication.
4. Let S be the length of the sidereal year, T be the length of the tropical year, μ be the daily solar motion, and p the annual increment in stellar longitudes due to precession. Then:

   $$p = \mu \cdot (S - T).$$

   For Ptolemy, $\mu \cdot T = 360°$. The parameter for precession of 1°/100 years is equivalent to 0;0,36°/y, and the daily solar motion is 0;59,8°. Hence $p/\mu = 0;0,36,32^d$ which, when added to $365;14,48^d$, yields a sidereal year of $365;15,24,32^d$.
5. *Cf.* O. Pedersen, *A survey of the Almagest* (Odense, 1974), 423. Note that Rheticus (*Narratio prima* (ref. 1), 102) gives the length of the sidereal year as about $365;15,24^d$.
6. *Cf.* Rheticus, *Narratio prima* (ref. 1), 95.
7. B. R. Goldstein, "Levi ben Gerson's analysis of precession", *Journal for the history of astronomy*, vi (1975), 31–41 (espec. p. 36).
8. See R. Morelon, *Thābit Ibn Qurra: Oeuvres d'astronomie* (Paris, 1987), p. lii.
9. See O. Neugebauer, "Thābit Ben Qurra 'On the solar year' and 'On the motion of the eighth sphere'", *Proceedings of the American Philosophical Society*, cvi (1962), 264–99 (espec. p. 264).
10. Morelon, *op. cit.* (ref. 8), 56; the Latin version has 43 instead of 33 in the last place: *cf.* Neugebauer, *op. cit.* (ref. 9), 280.
11. Morelon, *op. cit.* (ref. 8), 55, 205; Neugebauer, *op. cit.* (ref. 9), 279.
12. Morelon, *op. cit.* (ref. 8), 60; Neugebauer, *op. cit.* (ref. 9), 285.
13. F. J. Ragep, *Naṣīr al-Dīn al-Ṭūsī's* Memoir on astronomy (New York, Berlin, 1993), 400ff; Morelon, *op. cit.* (ref. 8), p. xix.
14. See Neugebauer, *op. cit.* (ref. 9); B. R. Goldstein, "On the theory of trepidation", *Centaurus*, x (1965), 129–60; J. Dobrzycki, "Teoria precesji w astronomii średniowiecznej" [in Polish with English summary], *Studia i materiały*, C 11 (Wrocław, 1965), 3–47; R. Mercier, "Studies in the medieval conception of precession", *Archives internationales d'histoire des sciences*, xxvi (1976), 197–220; J. Samsó, *Las ciencias de los antiguos en al-Andalus* (Madrid, 1992), 222ff.
15. G. J. Toomer, "A survey of the Toledan Tables", *Osiris*, xv (1968), 5–174 (espec. pp. 118 ff).
16. See J. Chabás and B. R. Goldstein, "Andalusian astronomy: al-Zīj al-Muqtabis of Ibn al-Kammād", *Archive for history of exact sciences* (in press).
17. M. Rico y Sinobas, *Libros del saber de astronomía del rey Don Alfonso X de Castilla* (5 vols, Madrid, 1863–67), iv, 179; *cf.* J. Samsó, "Algunas notas sobre el modelo solar y la teoría de la precesión de los equinoccios en la obra astronómica de Alfonso X", *Acta hispanica ad medicinae scientiarumque historiam illustrandam*, iv (1984), 81–114 (espec. p. 101).
18. E. Poulle, *Les tables alphonsines* (Paris, 1984); *cf.* J. D. North, "The Alfonsine tables in England", in *Prismata: Festschrift für Willy Hartner*, ed. by Y. Maeyama and W. G. Salzer (Wiesbaden, 1977), 269–301.
19. E. Poulle, "The Alphonsine tables and Alphonso X of Castille", *Journal for the history of astronomy*, xix (1988), 97–113; for some evidence that suggests otherwise, see now B. R. Goldstein, J. Chabás, and J. L. Mancha, "Planetary and lunar velocities in the Castilian Alfonsine Tables", *Proceedings of the American Philosophical Society*, cxxxviii (1994), 61–95.
20. *Cf.* J. Casanovas, "On the precession problem in the Alfonsine tables", in *De astronomia Alphonsi Regis*, ed. by M. Comes, R. Puig and J. Samsó (Barcelona, 1987), 79–87.
21. See D. J. Price, "A medieval footnote to Ptolemaic precession", *Vistas in astronomy*, i (1955), 66.
22. See R. Mercier, "Studies in the medieval conception of precession", *Archives internationales*

d'histoire des sciences, xxvii (1977), 33–71 (espec. pp. 58–59).

23. E. Poulle, "Jean de Murs et les tables alphonsines", *Archives d'histoire doctrinale et littéraire du moyen âge*, xlvii (1980), 241–71.
24. In another passage (Poulle, *op. cit.* (ref. 23), 262), John of Murs attributes this value to "Alzophi et alii" ("al-Ṣūfī and others"), and attributes the value of 1°/66y to al-Battānī, which is in fact correct.
25. Poulle, *op. cit.* (ref. 23), 257–8.
26. Poulle, *op. cit.* (ref. 23), 258n.
27. J. D. North, "Just whose were the Alfonsine tables?", *Proceedings of the Fifth International Symposium on the History of Arabic Science, Granada 1992* (in press).
28. Poulle, *op. cit.* (ref. 23), 251.
29. North, *op. cit.* (ref. 27).
30. Poulle, *op. cit.* (ref. 23), 258.
31. Poulle, *op. cit.* (ref. 23), 262.
32. *Almagest*, vii.3 (transl. by G. J. Toomer, *Ptolemy's Almagest* (Berlin, New York, 1984), 338); C. Nallino, *Al-Battānī sive Albatenii Opus astronomicum* (3 vols, Milan, 1899–1907), i, 124, 292.
33. "He [Alfonso] determined [by observation] in his time the positions of the fixed stars, which he compared with their positions as accurately (*veraciter*) found by the ancients; he divided the time between them ... and he found the amount of time [it took them] to move one degree; ... in this way he found 1;22,40,53° in 100 years, and this motion is 0;0,49,28° per year ..." (for the Latin text, see Poulle, *op. cit.* (ref. 23), 259). Is this a report of what John of Murs read in his source, or is it his reconstruction of what lay behind the parameters he found there? Note that according to the Parisian Alfonsine tables an accumulated precession of 17;8° corresponds to about 1236 years (the interval from the presumed epoch of Ptolemy's star catalogue, A.D. 16 (see J. Samsó and F. Castelló, "An hypothesis on the epoch of Ptolemy's star catalogue", *Journal for the history of astronomy*, xix (1988), 115–20), to the epoch of the Alfonsine tables, A.D. 1252), but this yields an 'average' precession of about 0;0,49,54°/year, and not the value in John of Murs's text.
34. Toomer, *op. cit.* (ref. 15), 44.
35. *Cf.* North, *op. cit.* (ref. 27).
36. To be sure, it is Copernicus's geometrical argument that correctly ascribes the phenomenon of precession to the axis of the Earth, rather than to a motion of the stars. For further details, see K. P. Moesgaard, "The 1717 Egyptian years and the Copernican theory of precession", *Centaurus*, xiii (1968), 120–38; N. M. Swerdlow and O. Neugebauer, *Mathematical astronomy in Copernicus's De revolutionibus* (2 vols, New York, Berlin, 1984), i, 129 ff.
37. Mercier, *op. cit.* (ref. 22), 60.
38. See J. M. Millás, *Las tablas astronómicas del rey Don Pedro el Ceremonioso* (Madrid, Barcelona, 1962), 190; J. Chabás, "Astronomia Hispanomusulmana: Las tablas de Barcelona" (in preparation: to appear in a volume edited by J. Samsó).
39. MS Vienna 5311 contains a number of astronomical treatises, including the *Theorica planetarum* by Campanus of Novara, and the *explicit* of that text (f. 100r) is dated 13 Sept. 1356 (F. S. Benjamin, Jr, and G. J. Toomer, *Campanus of Novara and medieval planetary theory* (Madison, 1971), 98, 355). The latest date in the MS is 1428, and it appears in marginal comments above the headings of star lists on ff. 129v, 130r, and 131r; the Alfonsine tables are mentioned on f. 134va (last line).

    On f. 137r there are reports of some solar and stellar altitudes observed in Bologna in 1305–6, Montpellier in 1306, and Genoa in 1311–12, and we are also told that the longitude of Spica in 1306 was Lib 12;50°. It is then remarked that the corresponding entry for Spica in Ptolemy's star catalogue is Vir 26;40°, which leaves a difference of 16;10°. According to this text, the epoch of Ptolemy's star catalogue is A.D. 141 (rather than A.D. 137 as in Toomer, *op. cit.* (ref. 32), 340; *cf.* North, *op. cit.* (ref. 27), n. 23), and this is 1165 years prior to 1306. The author of this note then divides 16;10° by 1165 years, and he finds that 1° corresponds to 72 years "and a little more" (accurately 1°/72;4 years). He goes on to say that two ancient observations of Spica, by Timocharis, were separated by 12 years, and the increase in longitude between them was 0;10° (as indeed we read in *Almagest,* vii.3; Toomer, *op. cit.* (ref. 32), 336) which, he adds, is equivalent to 1°/72 years. Though it is not explicitly stated, the point would seem to be that precession has not changed since the earliest available observations that go back to the third century B.C.

40. See Swerdlow and Neugebauer, *op. cit.* (ref. 36), i, 147.
41. See J. Dobrzycki, "Astronomical aspects of the calendar reform", in *Gregorian reform of the calendar*, ed. by. G. V. Coyne, S.J., M. Hoskin, and O. Pedersen (Vatican City, 1983), 117–37 (espec. pp. 120 ff); Dobrzycki, *op. cit.* (ref. 14).
42. Rheticus gives a list of the accumulated precession according to Copernicus for various epochs, but does not compare these values with calculations based on the Alfonsine tables (see transl. by Hugonnard-Roche *et al.* (ref. 1), 95). For such a comparison, see the graph in Mercier, *op. cit.* (ref. 22), 48. By way of contrast, Rheticus does compare Copernicus's model for the motion of the solar apogee with the Alfonsine tables for this motion, arguing that Copernicus's model shows better agreement with what he takes to be observational data by various practitioners from Hipparchus to Copernicus (transl. by Hugonnard-Roche *et al.* (ref. 1), 99–100, and 157, notes 56, 57).