Levi ben Gerson's Theory of Planetary Distances

by

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I. Introduction

The *Astronomy* of Levi ben Gerson (d. 1344) comprises 136 chapters and culminates with a discussion of planetary distances in which the arguments and computational procedures depart from those generally accepted in his time. In a series of earlier publications I have described other aspects of Levi's contributions to astronomy, including his original lunar theory and the astronomical instruments he invented (Goldstein 1985, and other publications listed there), leaving for the future his planetary theories and his determination of planetary distances. As long ago as 1969 I noted that Levi's value for the distance to the fixed stars, 157×10^{12} earth radii, is very much greater than the standard value in Ptolemy's *Planetary Hypotheses* of 20,000 earth radii that was accepted by most medieval astronomers (Goldstein 1969). At that time I did not understand Levi's underlying theory, and it is presented here for the first time. For a recent survey of astronomical theories of planetary distances from antiquity to the early modern period, see Van Helden 1985.

Chapter 130 of this treatise presents a general introduction to the principles governing his procedure which are then applied in chapter 131 (see the translations of these chapters that follow: note that references here to the translation are given by chapter and sentence number separated by a colon). As is often the case in Levi's *Astronomy,* the calculations give a clearer sense of his intent than the prose account does, and for that reason the two chapters should be read in

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conjunction. The structure of the cosmos, according to Levi, consists of planetary shells separated by fluid layers with certain properties that allow us to compute their thicknesses. The physical properties of this fluid are discussed in chapter 130 and are posited on analogies with terrestrial physics. So, we are told that "the strength of the impulse depends upon the strength of the motion" (130:14), and for "the throwing of stones, ... the greater the strength, the greater is the amount of the medium that receives the form of the impulse" (130:15). These principles are then applied to determine the effect of the motion of celestial spheres on the adjacent fluid layers. In cases where a sphere is eccentric we are told that it will generate turbulence in the fluid at its convex surface (130:25–26). The purpose of the fluid is to prevent the planetary motions from interfering with one another (130:41); this is accomplished by the diminution of motion in a layer of fluid as a function of its distance from the surface of a planetary sphere until a layer is reached where no motion is left. The impulse diminishes, and a weak impulse produces less motion until "due to the weakness of the impulse [a stone] will fall immediately upon its separation from the thrower" (130:16). This application of principles derived from terrestrial physics to celestial motions is certainly unusual in a medieval text and I know of no parallel to it.

Another unusual feature of chapter 130 is the treatment of comets that goes beyond the discussion in chapter 29. There Levi sought to show that the lowest orb of each planet is a concentric orb that moves with the daily rotation (Goldstein 1987). His most persuasive argument is that since comets are sublunary (according to the commonly accepted view in the Middle Ages) and are seen to participate in the daily rotation, they must receive this motion from the lowest lunar orb which must, therefore, be an orb that moves with the daily rotation. He then extends this to the other planets as well. In chapter 130 Levi reports his attempts to measure cometary parallax as a way to determine the thickness of the sublunary fluid that is moved by the daily rotation of the lowest lunar orb (130:5–10). It appears that he found no parallax ("this method did not yield the truth") and claimed that the reason for its absence was that "the nature of the celestial fluid is very different from the nature of the lower fluid which is near us" (130:10). Levi's application of Ptolemy's method for finding lunar parallax to comets is earlier than that of Regiomontanus (d. 1476), to whom the invention of this method has hitherto been attributed (cf. Jervis 1985, pp. 95–120). It was only in observing the comet of 1577 that Brahe showed comets to be free of measurable parallax and, hence, above the Moon. This new result contributed to the downfall of the Aristotelian distinction between the sublunary world of coming-into-being and passing-away, and the eternal unchanging heavens. In effect, Levi's observations were good, but his conclusion shows that he was still bound by the tradition that comets were sublunary.

Levi's theory of planetary distances depends on a number of stated and unstated assumptions:

(1) The relative maximum and minimum distances derived from his planetary models are in the same ratio as the true maximum and minimum distances.
(2) The maximum lunar distance has previously been found to be 62;48,42,36 terrestrial radii.
(3) The minimum solar distance has previously been found to be 2052;4,47 terrestrial radii.
(4) Each planet is moved by a set of orbs that consistute its spherical shell (or, simply, its sphere). The lowest part of this sphere moves with the daily rotation, and the highest part moves in the direction of the daily rotation with a velocity equal to the difference between 360° and the planet's slowest daily velocity to the east.
(5) The order of the planetary spheres from the Earth in chapter 131 is Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, and the Fixed Stars. Elsewhere he considers the Sun between Mercury and Venus, and the Sun below both Mercury and Venus.
(6) Between adjacent planetary spheres there is a superlunary fluid whose role is to make sure that none of the motion of one planet interferes with that of another.
(7) Below the moon is a sublunary fluid whose properties may differ from those of the superlunary fluid.
(8) There is no empty space in the cosmos.

Assumptions (1), (5), and (8) are taken from Ptolemy's *Planetary Hypotheses* (Goldstein 1967). However, Levi does not use Ptolemy's maximum and minimum distances because Levi introduced new models for planetary motions (see Table 1). The passages concerning plan-

	Ptolemy		
	Planetary Hyp.	Alm.	Levi
Mercury	$\frac{88}{34}$ (a)	$\frac{91;30}{33;4}$	$\frac{88;54,28}{36;\ 5,32}$
Venus	$\frac{104}{16}$	$\frac{104;25}{15;35}$	$\frac{104;45}{15;15}$
Mars	$\frac{7}{1}$	$\frac{105;30}{14;30}$	$\frac{104;21,52}{15;38,\ 8}$
Jupiter	$\frac{37}{23}$	$\frac{74;15}{45;45}$	$\frac{73;28,36}{46;31,24}$
Saturn	$\frac{7}{5}$	$\frac{69;55}{50;\ 5}$	$\frac{69;18,52}{50;41,\ 8}$

(a) With the parameters in the *Planetary Hypotheses,* one should find: 90;15 to 33;49 (Goldstein and Swerdlow 1970, p. 141).

Table 1. Relative Planetary Distances.

etary theory are incomplete in all surviving manuscripts of Levi's *Astronomy* and it is hoped that enough remains to reconstruct it. The use of epicycles is avoided by Levi because he claimed that an epicycle for the Moon would imply that we see both sides, contrary to experience, and he extends this argument to the planets as well. Levi presents specific dated observations on the basis of which he builds his planetary theories. On the matter of the order of the planetary spheres, Levi takes up the three variants that had been discussed in the twelfth century: Ptolemy's order with the Sun between Venus and Mars (in chapter 131); Jābir ibn Aflaḥ's order with the Sun below both Mercury and Venus (in chapter 134); and al-Biṭrūjī's order with the Sun between Mercury and Venus (in chapter 135). Levi notes that if Mercury and Venus were below the Sun, their parallax should be perceptible. Since no parallax has been observed, they probably lie above the Sun (chapter 133; P 252b:22–24). Other arguments are cited that tend toward the same conclusion (chapter 133).Nevertheless, Levi does a complete set of calculations for each of the three hypotheses while keeping his other assumptions unchanged.

Assumption (2) depends on Levi's lunar model in chapter 71 (Goldstein 1974a, pp. 53–74) and on his discussion of lunar distances in chapter 92 where this maximum lunar distance is derived. Assumption (3) is stated in 131:47 and depends on a passage in chapter 128 (Q 217a:16) where this minimum solar distance is stated. In chapter 128 we are referred to chapter 91, but there the minimum solar distance is given as 2061;4,25,40,7,55 terrestrial radii (Q 135b:16), a value based on an analysis of the lunar eclipse of 3 October 1335. However, in solar chapter 100, Levi tells us that he revised his value for the eccentricity based on his observation of the solar eclipse of 3 March 1337 (Goldstein 1979, pp. 104, 118) and that presumably accounts for the change in value of the maximum solar distance.

Assumption (4) departs from the tradition by making the orb of the daily rotation the lowest of each planet's orbs. In chapter 60 (P 111b), Levi investigates the solar sphere and tells us that three orbs suffice. The lowest is concentric with the earth and partakes in the daily rotation. The second is the orb of apogee that moves at the rate of 1° in $43\frac{2}{3}$ years about the poles of the ecliptic (cf. Goldstein 1978, pp. 46–47) and is eccentric to the center of the earth. The third orb is concentric with the second orb and moves with the mean motion. The Sun then moves with the combined motions of all three orbs. In the *Planetary Hypotheses,* Book II, Ptolemy also has three orbs for the Sun. But there the orb of the daily rotation is the highest one; and the intermediate orb, the eccentric orb of apogee, is fixed with respect to the ecliptic (Neugebauer 1975, p. 924; for the figure see p. 1403).

Although each of the other planets has more orbs than the Sun, the same general structure is manifest in all of them. Indeed, we are told that for the planets, the outermost orb is eccentric (cf. 130:25). The lowest orb that moves with the daily rotation is called the concave surface of the planet's sphere. The highest part of the planet's sphere is called its convex surface. If we disregard the daily rotation, the Sun's slowest motion takes place at apogee. To find the motion at the concave surface we must add the daily rotation to it algebraically (i.e., taking into account the direction of motion in deciding whether to add or subtract). For the planets Levi considers the greatest retrograde motion and adds the daily rotation to it to arrive at the planet's motion at its concave surface (cf. 130:34–36). In effect, the greatest retrograde motion to the west is the slowest motion to the east (a negative quantity in this case) and so we have the same rule for the planets that we had for the Sun and the Moon, namely, the velocity at the concave surface is 360° less the slowest daily velocity to the east (cf. 130:35).

Assumption (6) is not found in the antecedent tradition, although a passage in Ptolemy's *Planetary Hypotheses,* Book II, might be taken to hint at this view. Ptolemy had not filled the cosmos with planetary spheres; rather, he used "disks" cut from such spheres whose thickness depends on the planet's maximum latitude (cf. Hartner 1964, pp. 279–80) and, "for the transmission of the daily rotation to all stars and planets he postulates shells (of unspecified thickness) of a mysterious 'ether' between the contiguous planetary spheres" (Neugebauer 1975, p. 923). No numerical values occur in Book II, and the text is by no means entirely clear. However, in Book I of the *Planetary Hypotheses,* Ptolemy clearly states the nesting hypothesis according to which the maximum distance of one planet is equal to the minimum distance of the planet above it such that there is no empty space between them. Levi does not allow an orb of one planet to touch an orb of another planet because the motion of one planet would then affect that of the other. His interplanetary fluid is left over from before the creation of the universe (Freudenthal 1986; cf. Staub 1982, pp. 95–100 *et passim*; and Touati 1973, pp. 254–55). Levi was certainly aware of a passage in Maimonides *Guide of the Perplexed* (II.24) where the following view is presented:

> ... it follows necessarily that when the higher sphere is in motion it must move the sphere beneath it with the same motion and around its own center. Now we do not find this is so ... Hence necessity requires the belief that between every two spheres there are bodies other than those of the spheres ... Thābit [ibn Qurra, d. 901] has explained this in a treatise of his and has demonstrated what we said, namely, that there must be the body of a sphere between every two spheres ..." (trans. Pines, pp. 324–25).

Again, no numerical data are presented that would allow us to compute planetary distances (cf. Maimonides, *Guide,* III.14; trans. Pines, p. 457). I cannot find the view ascribed here to Thābit in any of his extant works; indeed, in his *Tashīl al-majistī* Thābit states Ptolemy's nesting hypothesis and uses it for computing his planetary distances (see Carmody 1960, p. 137). But Albertus Magnus cites a passage from Thābit's lost treatise, *Libro de Excentricitate orbium,* that concerned the nature of the matter constituting the celestial spheres. Here we are told that there is a subtle matter that fills the space be-

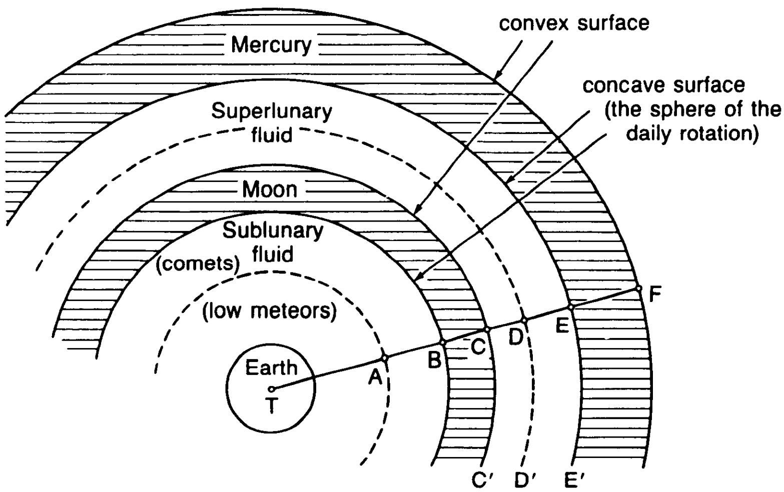

Figure 1. Note that eccentricities are not displayed.

tween the spheres. This matter is uniform, transparent, and subject to division (i.e., fluid), but not to alteration (Henquinet 1935, p. 285; cf. Morelon 1982, p. 430). Levi presumably depended on the passage in *The Guide* and not on Thābit directly, either in Arabic or in Latin.

Levi's approach to determining the thickness of his celestial fluid was to make certain arbitrary assumptions and then to use an iterative procedure to adjust some of these assumptions. We are told that there is no relative motion at the concave or convex surface of a planet's sphere, i.e., the fluid there moved at the same rate as that surface. As the radial distance from that surface increases, the motion of a layer of fluid at that distance diminishes uniformly until it reaches zero. The layer of zero motion is then assumed to have minimal thickness (130:40–41), since that is sufficient for its function (in practice, Levi considers this layer to have zero thickness). In Figure 1, the spherical shells (or spheres) of the Moon and Mercury are displayed: CC' represents the convex surface of the Moon and EE' represents the concave surface of Mercury. Between them is the celestial fluid and its motionless layer is represented by DD'. As the distance from the center of the Earth increases from TC to TD, the motion of the fluid decreases uniformly from the motion of the convex lunar surface to zero.

As the distance from the center of the Earth increases from *TD* to *TE,* the motion of the fluid increases uniformly from zero to that of the concave surface of Mercury. Levi's goal is to compute the thickness of the fluid, i.e. the sum of *CD* and *DE*. To do so he makes the following initial assumptions:

(a1) If the surface of a sphere moved with the daily rotation (i.e., $360^{\circ/\mathrm{day}}$, the motion in the fluid diminishes uniformly to zero in a layer of 6;40 in the measure where the radius of that surface is taken to be 60 (131:3).

(a2) If the motion of the surface of a sphere is not that of the daily rotation, the ratio to 6;40 of the thickness of the fluid layer in which the motion is reduced to zero is equal to the ratio of that motion to the daily rotation (where the radius of the surface is taken to be 60).

(a3) If the true distance of a planetary surface has been found in terrestrial radii, the thickness of the adjacent fluid layer can be translated into these units by simple proportions.

Levi does not tell us how he arrives at 6;40 as his initial value for the thickness of the fluid layer in assumption (a1): for a reconstruction of his procedure, see the commentary on 131:21–24. The name for his method is here translated "heuristic reasoning" (*heqqesh taḥbuli*); it is frequently used by Levi (though not by others), and nowhere clearly explained (see 130:37, 131:2, 131:2; cf. Goldstein 1985, p. 134; Goldstein 1974a, p. 67; Goldstein 1974b, p. 285). However, from its use in 131:24, we can see that it refers to an iterative procedure for adjusting parameters. In chapter 47 (P 93a:6–7) Levi uses this term to describe Ptolemy's method for finding the eccentricity and apogee of each outer planet. In the Hebrew translation of Averroes's *Epitome of the Almagest,* we find the term *mofet taḥbuli* (MS Oxford Opp. Add. fol. 17 [Neugebauer 2011], 122b:7–8) which is applied there to Ptolemy's method for finding the eccentricity and apogee of each outer planet (cf. Goldstein 1985, p. 134), and this may be one source for Levi's expression. Indeed, Levi wrote supercommentaries on many of Averroes's commentaries on Aristotle (Touati 1973, pp. 72–75).

In chapter 49 (P 94b:8) Levi attempts to explain "heuristic reasoning" saying that it is a kind of hypothetical syllogism (*heqqesh tena'i*) that involves a process of experiment and investigation (*nisayon ve-ḥippus*): this seems to be a rendering of an expression in the *Almagest,* IX.1: "If we were at any point compelled by the nature of our subject to use a procedure not in strict accordance with theory ... or if we were compelled to make some basic assumptions which we arrived at not from some readily apparent principle, but from a long period of *trial and application* ..." (trans. Toomer 1984, p. 422, emphasis added). Sabra (1971) points to another passage in the *Almagest* (Vii.1, trans. Toomer 1984, p. 325) where Isḥaq ibn Ḥunayn translated *synkrisis* (comparison) by the Arabic term *muqāyasa,* and *peira* by two Arabic words: *al-miḥna* (trial, test) and *al-iʿtibār* (consideration, experience). The Hebrew *heqqesh* corresponds to Arabic *muqāyasa* and the Hebrew *nisayon ve-ḥippus* corresponds to *al-miḥna wal-iʿtibār.* Unfortunately, the Hebrew translation of the *Almagest* does not use Levi's expressions in these places (cf. MS Paris, B. N., heb. 1018), but they may have reached Levi indirectly.

The explanation of "heuristic reasoning" is to be found in the *Compendium of the Sciences* by al-Fārābī (d. 950) where the category *ʿilm al-ḥiyal* is defined. Roughly translated this is the science of devices (*ḥiyal* is the plural of *ḥīla* that corresponds to the Hebrew *taḥbula*). In effect, al-Fārābī introduces a distinction other than the usual one between the theoretical and practical sciences, for this "science of devices" concerns the ways of manipulating natural objects (both physical and mathematical) to make them serve a useful purpose, and reflections upon them. It is this two-fold aspect of *ḥīla,* i.e. manipulation and reflection, that Levi seeks to capture by the expression "experiment and investigation". According to al-Fārābī, this science includes algebra (where the goal is to find a number that solves an equation); geometric proportions (where the goal is to find an unknown quantity in terms of certain given quantities); the preparation and use of mechanical devices, musical instruments, mirrors, etc. (A. Gonzales Palenncia 1953, pp. 154–156 [Latin], 73–76 [Arabic]). Though in Latin this science is later understood as engineering (*ḥīla* is translated as *ingenium*), al-Fārābī clearly wishes it to apply to a domain larger than that. This category of science differs from a theoretical one because the reasoning is *heuristic* rather than deductive, and the practitioner may not have a theory to apply; rather, he combines reflection with experience (for the importance of this passage in al-Fārābī, see Rashed 1975, p. 483; a Hebrew version of it, using the term *ḥokhmat ha-taḥbulot,* was made in the thirteenth century by Shemtov Ibn Falaquera and appears in his *Reshit Ḥokhma*). This concept deserves more

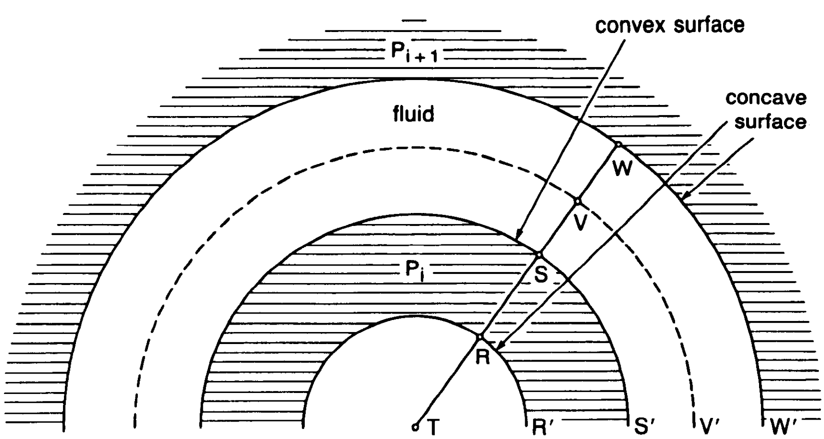

Figure 2. Note that eccentricities are not displayed.

detailed analysis, but that must be postponed to another occasion. Here let me only add that in the encyclopedia of al-Khwārizmī (fl. ca. 975) we are told that "the science of *ḥiyal*" is called by the Greeks "mechanics" (Wiedemann 1970, 1:190), a traditional identification from which al-Fārābī departed.

Levi computes the fluid layers between the Moon and Mercury, Mercury and Venus, and Venus and the Sun. With the use of 6;40 in assumption (a1) he did not arrive exactly at the minimum solar distance and so the 6;40 is changed to 7;11 (131:25) and again to 7;9,20 (131:42), each time diminishing the discrepancy with the previously determined minimum solar distance. Once he has settled on the value 7;9,20 he is ready to compute the distances of the outer planets and the fixed stars using the same procedures. We thus see that Ptolemy's nesting hypothesis has been modified to take the fluid into account, and that the fluid serves to answer Maimonides's attack (cited above) on this aspect of Ptolemy's theory.

To illustrate Levi's method, let us consider Figure 2 where P_i is the spherical shell (or sphere) of Mars and P_{i+1} is the spherical shell (or sphere) of Jupiter. Note that between the spherical shells is a fluid layer, and that each spherical shell is composed of orbs which are not shown here. The value of *TW*, Jupiter's minimum distance from the center of Earth, is to be calculated from the minimum distance of

Mars's sphere, *TR*, which was previously found to be 2832;56 t.r. (terrestrial radii). We first must find the thickness of Mars's sphere and then the thickness of the fluid layer between Mars and Jupiter (131:55–60). To solve the first problem we use the ratio of maximum to minimum distance (*D*/*d*) in Levi's model for Mars (this model is the subject of chapters 117–120: unpublished), and assume that the distances in terrestrial radii are in the same ratio. Hence

$$\frac{D}{d} = \frac{TS}{TR} \tag{1}$$

or

$$\frac{104;21,52}{15;38,8} = \frac{TS}{2832;56 \text{ t.r.}} \tag{2}$$

and so the distance to the convex surface of Mars's sphere

$$TS = 18909;18 \text{ t.r.} \tag{3}$$

as mentioned in 131:56. This value has been accurately computed by Levi: where recomputation differs by more than a rounding error, the accurate value will be indicated in parentheses following the value in the text. The thickness of the fluid layer between Mars and Jupiter, *SW*, is found in two steps. The first step yields the distance *SV*, and the second step yields the distance *VW*, where *TV* is the distance to the motionless layer in the fluid between Mars and Jupiter. Where Levi uses units other than terrestrial radii, we will use lower case letters to indicate the distances in Figure 2. First we must find the greatest retrograde motion of Mars. Levi takes this to be $1;9^{\circ/\text{day}}$ without indicating here how this value was computed (see Table 2 for a list of his values for the maximum planetary retrograde motions). This motion is to be added to the daily rotation yielding $361;9^{\circ/\text{day}}$ for the greatest daily motion of Mars to the west, the motion at the convex surface of Mars's sphere. Then, by assumption (a2) with the value 7;9,20 (instead of the initial value of 6;40), we have

$$\frac{361;9^{\circ/\text{d}}}{360^{\circ/\text{d}}} = \frac{sv}{7;9,20} \tag{4}$$

Mercury	$0;45,16^{\circ/\text{day}}$	(131:10)
Venus	$0;44,27^{\circ/\text{day}}$	(131:15)
Mars	$1;9^{\circ/\text{day}}$	(131:56)
Jupiter	$0;8,11^{\circ/\text{day}}$	(131:62)
Saturn	$0;4,38^{\circ/\text{day}}$	(131:68)

Table 2. Maximum Retrograde Motions.

and

$$sv = 7;10,42,17 \tag{5}$$

as in 131:57; this is the thickness of the fluid layer above Mars in which the motion is diminished to zero, where the radius of the convex surface of Mars's sphere is 60. In this measure, the distance from the center of the earth to the motionless layer tv, is 67;10,42,17. We now turn to the determination of the thickness of the fluid layer from this place to the concave surface of Jupiter. In the measure where the radius of the concave surface of Jupiter's sphere, tw, is 60, the radius of the motionless layer below it, tv, is 52;50,40 (= 60 −7;9,20). It is Levi's view that this concave surface moves with the daily rotation. Thus

$$\frac{(tv)_1}{(tv)_2} = \frac{(tw)_1}{(tw)_2} \tag{6}$$

where the subscript 1 refers to the measure where the radius of the convex surface of Mars's sphere, ts, is 60; and subscript 2 to the measure where the radius of concave surface of Jupiter's sphere, tw, is 60. Thus

$$\frac{67;10,40,17}{52;50,40} = \frac{(tw)_1}{60} \tag{7}$$

and so

$$(tw)_1 = 76;16,30 \tag{8}$$

as in 131:59. If we now set the distance to the convex surface of Mars's sphere, TS, equal to 18909;18 t.r. as in (3), and substitute the appropriate values in (9),

$$\frac{tw)_1}{(ts)_1} = \frac{TW}{TS} \tag{9}$$

we have

$$\frac{76;16,30}{60} = \frac{TW}{18909;18 \text{ t.r.}} \tag{10}$$

and so

$$TW = 24038;27 \text{ t.r.} \tag{11}$$

as mentioned in 131:60; and this is the minimum distance of Jupiter that we sought. If we reconsider this procedure, we may note that the minimum distance of Jupiter was found from the minimum distance of Mars by compounding two ratios: the ratio of maximum to minimum distances of Mars in its model (D/d), and the ratio of the distances of the outer edge to the inner edge of the fluid between Mars and Jupiter (tw/ts). Thus

$$TS = \frac{tw}{ts} \cdot \frac{D}{d} \cdot TR. \tag{12}$$

Despite the complex rules for determining the ratio of tw to ts, this ratio remains fairly constant for all the interplanetary fluid layers and depends primarily on the initial value taken in assumption (a1): see 131:23, and the commentary on it.

Table 3 displays Levi's values for the maximum and minimum planetary distances. Clearly the other two planetary orders yield far higher values for the distance to the fixed stars because there will be far more fluid between the Moon and the Sun, and this in turn regulates the thickness of the fluid layers beyond the Sun. Indeed, with the Sun below both Venus and Mercury, Levi begins by assuming a fluid layer of 56;29,24 (chap. 134, P 254a:3) instead of 6;40 (131:3) and finds the distance to the fixed stars to be more than 157×10^{12} t.r. (chap. 134; P 255a:29, Q 225:8; cf. Goldstein 1974a, p. 29). Ptolemy's 20000 t.r. to the sphere of the fixed stars seem quite small in comparison with Levi's enormous cosmic distances. For a derivation of Levi's distance to the fixed stars, see the commentary on 131:67–72.

	Minimum distance	Maximum distance
Moon		62;48,42,36 t.r.
Mercury	79;32 t.r.	184;54
Venus	235; 2	1614;23
Sun	2052; 4,47	2229;29
Mars	2832;56	18909;18
Jupiter	24038;27	37965; 9
Saturn	48248;15	65981;21
Fixed Stars	83851;18	

Table 3. Levi ben Gerson's Planetary Distances in Chapter 131.

II. Note to the translation

For these two chapters we have only two manuscript copies:

P. Paris, Bibliothèque Nationale, heb. 724

Q. Paris, Bibliothèque Nationale, heb. 725.

The Hebrew text of chapter 130 appears in P 247a:3 to 248:15~~and in Q 218a:15~~ and in Q 218a:15 to 219a:16; and chapter 131 in P 248:15 to 250b:24 and in Q 219a:16 to 221b:13. For a description of these manuscripts, see Goldstein 1974a, pp. 74–79; no copy of the Latin translation contains these two chapters.

This translation is based on an unpublished edition that I have prepared: sentence numbers in square brackets have been added, and variants in the numerical data as well as explanatory remarks are also enclosed in square brackets. In the Introduction numbers separated by a colon refer to the translation by chapter and sentence.

III. Translation

Chapter 130

[1] It is clear that when Venus and Mercury are put below the Sun, it is possible for us to determine the distance between the spheres of one planet and another such that some fluid [*lit.*: the body which does not preserve its shape]

will be between them in which the motion of spheres touching it terminates (*tikhleh*) in such a way that there remains in its midst a motionless (*qayyam:* fixed) thing. [2] We decided to present here some useful premises for this investigation.

[3] The first is that the solid [*lit.:* the body which preserves its shape] moves the fluid below it with the likeness (*dimyon*) of its motion up to a certain amount, and the eye sees this revealed in the lowest sphere of the Moon, the sphere of the daily rotation, that moves, with the likeness of its motion, the [upper] part of the [sublunary] elements that accompanies it, as is seen in the motion of comets [*lit.:* tailed stars] and the like, that move with the daily rotation. [4] It has been found that the low meteors do not move with this motion, and this may serve as a sign that this impulse (*hanaᶜa*) has a definite limit for fluids.

[5] We found an approach [*lit.:* opening] by which, with lengthy examination, we may determine the amount by which the fluid below it moves with this impulse. [6] It may be possible for us to determine the amount of the distance of these meteors that move with the daily rotation from the center of the earth. [7] In that case, since the distance from the concavity (*qibbuv*) of the Moon's sphere of apogee to the center of the earth is already known, it would be clear how far this impulse is carried into the fluid. [8] But how may the distance of these tailed meteors [i.e., comets] from the center of the earth be found? [9] I say that we observe this meteor on one night when it is at different zenith distances, for from these we may determine the parallax of this meteor from which we can determine its distance from the earth by means of the same procedure we used for the Moon. [10] But when we embarked on this investigation, we found that this reasoning (*ha-heqqesh*) did not yield the truth because the nature of the celestial fluid is very different from the nature of the lower fluid which is near us [i.e. sublunary].

[11] Therefore, it is not appropriate for us to take the amount from this to that and, in general, if this reasoning had yielded the truth in this, it would have been possible to explain from it that it is impossible that Venus and Mercury lie below the Sun, for that distance would not suffice for the distance required between the spheres of one planet and the other on this assumption. [12] This will be clear to you if you investigate it in the way mentioned, but this reasoning does not work for the reason we mentioned. [13] The appropriate approach, by which we may determine this, is by means of the excess in the distance that we found between the spheres of the Sun and the spheres of the Moon, over what is appropriate for taking into account the distance required for the spheres of Mercury and Venus.

[14] The second is that the strength of the impulse depends upon the strength of the motion. [15] This is clarified from the throwing of stones, for the greater the force, the greater is the amount of the medium (*ha-emṣaᶜi*)

that receives the form of the impulse. [16] But when the [force] is weak, its amount is diminished until it happens, due to the weakness of the impulse, that it [i.e., the stone] will fall immediately upon its separation from the hand of the thrower. [17] From this it is clear that the daily spheres move the celestial fluids below them with differing amounts. [18] This is because the sphere whose concavity is greater moves with greater strength, for its motion is stronger. [19] This is because the ratio of the size of one of these concavities to the other is equal to the ratio of their motions because these motions are completed in the same amount of time. [20] Therefore, what moves the fluid below it ought to be greater according to the ratio of one size to the [other] size. [21] It is clear that if we measure the amount that the fluid moves in units of the distance from the concavity of the daily sphere to the center of the earth, always setting the radius at 60, the different amounts will be in the ratio of the different motions.

[22] The third is that just as the sphere moves the fluid below it, so it moves the fluid above it up to a certain amount, but it is more appropriate in this for two reasons. [23] One is that the impulse is stronger for the higher than for the lower, i.e., something that has the force to move the solid above it, may not have the force to move the solid below it. [24] This reasoning is as follows: something, which has not the force to move the solid below it, may have the force to move the fluid below it; clearly that which has the force to move the solid above it, certainly has the force to move the fluid above it which is contiguous to it, but it seems appropriate that it be the stronger in this [case].

[25] The second reason is that since the convexity of the planetary spheres is eccentric to the earth, and the daily rotation takes place about the center of the earth, it is clear that by this motion it [i.e., the convex surface] disturbs the state (*maṣav*) of the fluid above it, and on account of it, the form of the impulse present in this fluid is strengthened. [26] Moreover, it disturbs the shape of the fluid continuously changing it to another shape, and this is what follows from the motion being there in many of the parts of that fluid. [27–28] This is because when the apogee (*govah*) of the convexity of the sphere of mean motion and the apogee of the convexity of the sphere of apogee lie in the same direction, the planet will be at its greatest distance, and the surface of the fluid which is at the surface of the motion of the planet varies in thickness (*ʿov*) as greatly as possible. [29] The thickness of the thicker side [of the spherical shell] may be greater than the thickness of the side opposite it by the sum of twice the eccentricity of the sphere of apogee and twice the eccentricity of the sphere of the mean motion.

[30] But when the apogee of the convexity of the sphere of the mean motion is in the direction of the perigee of the convexity of the sphere of the motion [*sic; read:* apogee], there will be a diminution in the excess of thickness

that we mentioned by double the eccentricity of the sphere of the apogee, twice, and the shape of the fluid differs in this place from the first shape varying in this way and producing innumerably many different shapes. [31] It also follows that some of the fluid moves because of the motion of the sphere of the planet below it. [32] From this, it is also clear and without doubt, that of necessity the fluid lies between the spheres of one planet and those of another in such a way that it is compelled to receive these different shapes.

[33] Since this has been established, it is clearly appropriate that there be [enough] fluid between the spheres of one planet and the spheres of another such that a motionless (*qayyam*) [layer] may remain in its midst to make sure that the motions are not confused. [34] Since some of this body moves with the motion of the concavity of the daily sphere above it, and some of it moves with the convexity of the sphere below it, it is appropriate that we consider the ratio of this amount of motion to that, for these motions are in the ratio of the motions one with respect to the other, and we consider the daily motion as the measure for determining the variations in the motions. [35] The apparent motion of the convexity of the planetary sphere takes place to the west, while the motion of the planet to the east is slower, either less than the daily motion or greater for planets which have an apparent retrograde motion. [36] Thus we take an amount, according to the ratio, of the fluid that moves with respect to the daily motion, in order to arrive at the maximum amount for the fluid that moves due to the motion of the convexity of the planetary sphere [below it]. [37] We consider our method in this investigation [to be sound] for determining these amounts because of the excess distance that we found between the Sun and the Moon, and we distribute it by means of the hypothesis of heuristic reasoning (*heqqesh taḥbuli*) until all the distances agree together with what was assumed for the distance between the Sun and the Moon, for we have no other device (*taḥbula*) but this one to determine these amounts, as we mentioned earlier.

[38] For each thing that we have [done], we should state the assumptions that led us to determine what knowledge we may have found, even though this knowledge is not of the kind that doubt cannot enter. [39] For this reason, we do not set a definite amount for the part in the middle of the fluid which does not move, for we cannot determine the truth of this matter. [40]
/ as if it were
Therefore, we assume that this motionless place is ~~is~~ imperceptibly small. [41] To serve as an example: it is sufficient for its thickness to be one span or less, and this is an imperceptible amount at this distance from us, and it should be the least possible in which this [termination of motion] may be completed, for it has no function except to prevent the motions from disturbing each other, and a small amount is sufficient for this.

[End Chapter 130]

* * *

Chapter 131

[1] Now we should examine, according to the preceding method, the amount that the fluid moves on account of the daily motion in the measure where the radius of its concavity is 60, for from this we may determine the rest of the distances of the planets from the center of the earth inasmuch as that is possible. [2] We will arrive at this knowledge by means of heuristic reasoning, experimenting with lesser and greater, until agreement is reached with the distance between the Sun and the Moon.

[3] First we assume, by way of setting the heuristic reasoning, that the amount of the part of the fluid adjacent (*lit.*: touching) to it that the concavity of the daily sphere moves is 6;40 in the measure where the radius of the concavity of that sphere is 60. [4] Since the slowest motion of the Moon to the east is 11;53,30°, its motion to the west, the complement in 360°, is 348;6,30°. [5] We take from the 6;40 according to the ratio of the daily motion, which is 360°, to the motion of apogee of the convexity of the sphere of the Moon which at most is 348;52,35°, and it is 6;26,30,29 in the measure where the distance of this apogee from the center of the earth is 60. [6] Since the part of the fluid that the concavity of the daily sphere of Mercury moves is 6;40 in the measure where the radius of that concavity is 60, the 66;26,30,29 will [correspond to] 53;20 in that measure. [7] If we multiply these 66;26,47,9 by 60 and divide the product by 53;20, the result is the distance of the concavity of the daily sphere of Mercury from the center of the earth in the measure where the apogee of the convexity of the sphere of the Moon from the center of the earth is 60, and it is 74;44,49. [8] In the measure where the distance of the apogee of the convexity of the sphere of the Moon from the earth is 62;48,42,36, the distance of the concavity of the sphere of Mercury from the center of the earth is 78;14,58, very nearly.

[9] Since the ratio of the apogee of the convexity of the sphere of Mercury to the apogee of the concavity of its daily sphere is equal to the ratio of 83;54,28 to 36;5,32, the apogee of the convexity of Mercury's sphere in this measure is 181;54,56. [10] Moreover, the greatest daily motion of Mercury to the west is 360;45,16°. [11] When we take an amount from the 6;40 according to the ratio of this motion to 360°, the amount for the fluid that is adjacent to the apogee of the convexity of Mercury's sphere is 6;40,52 in the measure where the distance of the apogee of the convexity of Mercury from the center of the earth is 60.

[12] Since the part of the fluid that the concavity of the daily sphere of Venus moves is 6;40 in the measure where the radius of the concavity of that sphere is 60, what is below it to the center of the earth is 53;20 in this measure. [13] If we multiply these 66;40,52 by 60 and divide the product by 53;20, the result is the distance of the concavity of the daily sphere of Venus from

the center of the earth in the measure where the distance of the apogee of the convexity of the sphere of Mercury from the center of the earth is 60, and it is 75;0,56 [P: 75;56, and Q: 78;0,56]. [14] In the measure where the apogee of the convexity of the sphere of Mercury is 181;54,57, the distance of the concavity of the sphere of Venus from the center of the earth is 227;26,27.

[15] Since the ratio of the apogee of the convexity of the sphere of Venus from the center of the earth is equal to the ratio of 104;45 to 15;15, the apogee of the convexity of the sphere of Venus over the center of the earth is 1562;15,27 in the measure where the radius of the earth is 1. [16] The greatest daily motion of Venus to the west is 360;44,27° [Q: 364;44,27°]: when we take an amount from the 6;40 according to the ratio of this motion to 360°, the amount of the fluid adjacent to it that the apogee of the convexity of the sphere of Venus moves is 6;40,49,23 in the measure where the distance of the apogee of the convexity of the sphere of Venus from the center of the earth is 60. [17] Since the part of the fluid that the concavity of the daily sphere of the Sun moves is 6;40 in the measure where the radius of the concavity of that sphere is 60, what is below it to the center or the earth is 53;20 in this measure. [18] If we multiply these 66;40,49,23 by 60, and divide the product by 53;20 the result is the distance of the concavity of the daily sphere of the Sun from the center of the earth in the measure where the distance of the apogee of the convexity of the sphere of Venus from the center of the earth is 60, and it is 75;0,56 [P: 75;56]. [19] In the measure [*lit.*: distance] where the apogee of the convexity of the sphere of Venus over the center of the earth is 1562;15,27 terrestrial radii, the distance of the concavity of the daily sphere of the Sun from the center of the earth will be about 1952;49,40. [20] But this is less than what is appropriate according to our earlier [result] by 99;15,7.

[21] It is clear that placing the spheres of Venus and Mercury between the Sun and the Moon without assuming the impulse (*hanaʿa*) of the fluid to be 6;40, the distance of the concavity of the daily sphere of the Sun would only reach 1003;2,36, as mentioned in Chapter 128 [P: 27] of this treatise. [22] When we assumed that the daily motion moves the part of the fluid adjacent to the daily sphere [to a depth of] 6;40, it led to a distance for the concavity of the daily sphere of the Sun of 1952;49,40 which is higher than this previous computation by 949;47, and the surplus of the [other] previous computation over it is a little more than 99;15. [23] It is clear that the ratios led us to add to the 1003;2,36 by taking the amount of the fluid as about the ratio of 4 to 5 cubed, because for each one of them the ratio of the distance of the convexity of the sphere of Mercury from the center of the earth to the distance of the concavity of the daily sphere above it, is equal to the ratio of 60 to 75, and so it is appropriate that we take the ratio cubed in the product that increases the 1003;2,36 over the amount produced by this ratio, the 99;15. [24] When we examined this in the way, we found by heuristic reasoning that it is appropri-

ate to consider the amount of the fluid adjacent to it that the concavity of the daily sphere moves with its motion to be 7;11 in the measure where the radius of the concavity of that sphere is 60.

[25] Since the greatest daily motion of the apogee of the convexity of the sphere of the Moon to the west is 347;52,35, we take from the 7;11 in the ratio of this motion to the daily motion of 360°, and it is 6;56,29,7. [26] This is the amount of the fluid adjacent to it that the apogee of the convexity of the sphere of the Moon moves in the measure where its distance from the center of the earth is 60. [27] Since the part of the fluid that the concavity of the daily sphere of Mercury moves is 7;11 in the measure where the radius of that concavity is 60, there remains below it to the center of the earth 52;49 in this measure. [28] If we multiply these 66;56,29,7 by 60 and divide the product by 52;49, the result is the distance of the concavity of the daily sphere of Mercury from the center of the earth in the measure where the distance of the apogee of the convexity of the sphere of the Moon from the center of the earth is 60, and it is 76;2,45. [29] But in the measure where the distance of the apogee of the convexity of the sphere of the Moon from the center of the earth is 62;48,42,36, the distance of the concavity of the daily sphere of Mercury from the center of the earth is 79;36,33, very nearly.

[30] Since the ratio of the distance of the apogee of the convexity of the sphere of Mercury from the center of the earth to the apogee of the concavity of its daily sphere is equal to the ratio of 83;54,28 to 36;5,32, the distance of the apogee of the convexity of the sphere of Mercury from the center of the earth in this measure is 185;4,36. [31] Also the greatest daily motion of Mercury to the west is 360;45,16°. [32] When we take from the 7;11 according to the ratio of this motion to the daily motion, the part of the fluid adjacent to it that the apogee of the convexity of the sphere of Mercury moves is 7;11,54,12 in the measure where the distance of the apogee from the center of the earth is 60. [33] Since the part of the fluid that the concavity of the daily sphere of Venus moves has the amount 7;11 in the measure where the radius of that sphere is 60, what is below it to the center of the earth, in this measure, is 52;49. [34] If we multiply these 67;11,54,12 by 60 and divide the product by 52;49, the result is the distance of the concavity of the daily sphere of Venus from the center of the earth in the measure where the distance of the apogee of the convexity of the sphere of Venus from the center of the earth is 60, and it is 76;20,20. [35] In the measure [*lit.:* distance] where the apogee of the convexity of the sphere of Mercury from the center of the earth is 185;4,36 [P: 181;4,36], the distance of the concavity of the daily sphere of Venus from the center of the earth is 235;28,33.

[36] Since the ratio of the distance of the apogee of the convexity of the sphere of Venus from the center of the earth to the distance of the concavity of its daily sphere from the center of the earth is equal to the ratio of 104;45

to 15;15, the distance of the apogee of the convexity of the sphere of Venus from the center of the earth will be 1617;26,56. [37] Moreover, the greatest daily motion of Venus to the west is 360;44,24. [38] When we take from the 7;11 according to the ratio of this motion to the daily rotation, the part of the fluid adjacent to it that the apogee of the convexity of the sphere of Venus moves is 7;11,53 in the measure where the distance of the apogee of the convexity of the sphere of Venus from the center of the earth is 60. [39] Since the part of the fluid that the concavity of the daily sphere of the Sun moves has the amount of 7;11 in the measure where the radius of this concavity is 60, there remains below it to the center of the earth the amount of 52;49 in this measure. [40] When we multiply these 57;11,53 by 60 and divide the product by 52;49, the result is the distance of the concavity of the daily sphere of the Sun from the center of the earth in the measure where the distance of the apogee of the convexity of the sphere of Venus from the center of the earth is 60, and it is 76;20,14. [41] But in the measure where the distance of the apogee of the convexity of the daily (*sic*) sphere of Venus from the center of the earth is 1617;26,56, the distance of the concavity of the daily sphere of the Sun from the center of the earth will be 2057;46,54, and this is about 5;42,47 greater than it ought to be.

[42] Since the addition of 31 minutes to the amount of the fluid that moves according to the daily motion added 104;18 to the distance of the concavity of the daily sphere of the Sun, it is clear according to the ratio, using heuristic reasoning, that a diminution of 0;1,40 from the 0;31 will diminish the distance of the concavity of the daily sphere of the Sun by these 5;47, i.e. there remains 7;9,20 for the amount of the fluid that moves according to the motion of the concavity of the daily sphere. [43] It is clear, according to the ratio, that under this assumption the distance of the concavity of the daily sphere of Mercury, in the measure where the radius of the earth is 1, is 79;32, very nearly. [44] In this measure the distance of the apogee of the convexity of the sphere of Mercury from the center of the earth is 184;54, very nearly. [45] Also, in this measure the distance of the concavity of the daily sphere of Venus from the center of the earth is 235;2, very nearly. [46] Further, in this measure the distance of the apogee of the convexity of the sphere of Venus from the center of the earth is 1614;23 [P: 1314;23], very nearly. [47] Finally, the distance of the apogee of the concavity of the daily sphere of the Sun from the center of the earth is 2052;4,47, very nearly.

[48] After these matters have been brought into agreement, we may continue with the distances of the remaining planetary spheres that are above the Sun. [49] With a little reflection on what was explained in Chapter 91 of this treatise, it is clear that the distance of the apogee of the convexity of the sphere of the Sun from the center of the earth in this measure is 2229;29 [Q: 2229;31], very nearly. [50] Further, the greatest daily motion of the Sun to

the west is 359;3,12. [51] When we take from the 7;57,20 [Q: 7;56,20] according to the ratio of this motion to the daily motion, it is clear that the part of the fluid adjacent to it that the apogee of the convexity of the daily (sic) sphere of the Sun moves is 7;8,12 [Q: 7;1,12] in the measure where the distance from the apogee of the convexity of the sphere of the Sun from the [center of the] earth is 60. [52] Since the part of the fluid that the concavity of the daily sphere of Mars moves has the amount 7;9,20 in the measure where the radius of that sphere is 60, what is below it to the center of the earth in this measure is 52;50,40. [53] When we multiply these 67;8,12 by 60 and divide the product by 52;50,40, the result is the distance of the concavity of the daily sphere of Mars from the center of the earth, namely, 76;14,24,26. [54] In the measure where the distance of the apogee of the convexity of the sphere of the Sun is 2229;29, the distance of the concavity of the daily sphere of Mars from the center of the earth is 2832;56, very nearly.

[55] Since the ratio of the distance of the apogee of the convexity of the sphere of Mars from the center of the earth to the distance of the concavity of the daily sphere of Mars is equal to 104;21,52 to 15;38,8, the distance of the apogee of the sphere of Mars from the center of the earth is 18909;18 [with P mg. & Q; P: 19909;14], very nearly. [56] Moreover, the greatest daily motion of Mars to the west when it is at its greatest distance i.e., at 180° on the sphere of apogee, is 361;9. [57] When we take from the 7;9,20 according to the ratio of this motion to the daily motion, the amount that the apogee of the convexity of the sphere of Mars moves the part of the fluid adjacent to it is 7;10,42,17 in the measure where the distance of the apogee of the convexity of Mars from the center of the earth is 60. [58] Since the part of the fluid that the concavity of the daily sphere of Jupiter has the amount of 7;9,20 in the measure where the radius of that sphere is 60, the amount below it to the center of the earth in this measure is 52;50,40. [59] When we multiply these 67;10,42,18 by 60 and divide the product by 52;50,40, the result is the amount of the distance of the concavity of the daily sphere of Jupiter from the center of the earth in the measure where the distance of the apogee of the convexity of the sphere of Mars is 60, namely, 76;16,30. [60] But in the measure where the apogee of the convexity of the sphere of Mars is 18909;18, the distance of the concavity of the daily sphere of Jupiter from the center of the earth is 24038;27.

[61] Since the ratio of the distance of the apogee of the convexity of the sphere of Jupiter from the center of the earth to the distance of the concavity of the daily sphere of Jupiter is equal to 73;28,36 [P: 73;28,37] to 46;31,24 [P: 47;31,24], the distance of the apogee of the convexity of the sphere of Jupiter from the center of the earth will be 37965;9 [P: 37968;9]. [62] Further, the greatest motion of Jupiter to the west when it is at greatest distance is

360;8,11°. [63] When we take from the 7;9,20 according to the ratio of this motion to the daily motion, the part of the fluid adjacent to it that the apogee of the convexity of the sphere of Jupiter moves has the amount of 7;9,28 [Q: 7;9,27].

[64] Since the part of the fluid that the concavity of the daily sphere of Saturn moves has the amount 7;9,20 in the measure where the radius of this concavity is 60, there remains below it to the center of the earth 52;50,40 in this measure. [65] When we multiply these 67;9,28 by 60 and divide the product by 52;50,40, the result is the distance of the concavity of the daily sphere of Saturn from the center of the earth in the measure where the distance of the apogee of the convexity of the sphere of Jupiter from the center of the earth is 60, namely, 76;15,5. [66] But in the measure where the distance of the apogee of the convexity of the sphere of Jupiter from the center of the earth is 37965;9, the distance of the concavity of the daily sphere of Saturn from the center of the earth is 48248;15.

[67] Since the ratio of the distance of the apogee of the convexity of the sphere of Saturn from the center of the earth to the distance of the concavity of the daily sphere of Saturn from the center of the earth is equal to 69;18,52 [P: 69;18,2] to 50;41,8, the distance of the apogee of the convexity of the sphere of Saturn from the center of the earth is 65981;21 [P: 65988;51]. [68] Further, the greatest motion of Saturn to the west where it is at the greatest distance is 360;4,38. [69] When we take from the 7;9,20 according to the ratio of this motion [to] the daily [motion], the part of the fluid adjacent to it that the apogee of the convexity of the sphere of Saturn moves is 7;9,25. [70] Since the part of the fluid that the concavity of the sphere of the fixed stars moves has the amount of 7;9,20 in the measure where the radius of this concavity is 60, there remains below it to the center of the earth 52;50,40 in this measure. [71] When we multiply these 67;9,25 by 60 and divide the product by 52;50,40, the result is the distance of the concavity of the sphere of the fixed stars from the center of the earth, namely, 76;15 in the measure where the distance of the apogee of the convexity of the sphere of Saturn from the center of the earth is 60. [72] But in the measure where the distance of the apogee of the convexity of the sphere of Saturn from the center of the earth is 65981;21 [P: 62981;21], the distance of the concavity of the daily sphere of the fixed stars from the center of the earth is 83851;18 in the measure where the radius of the earth is 1.

[73] The distance of the apogee of the convexity of the sphere of the fixed stars from the center of the earth is greater than this by not less than the size of the diameter of the largest star on it. [74] We shall determine the size of the stars needed here to explain that the distance of the stars is not less than what we mentioned, and it is possible for us to acquire that knowledge. [75]

Indeed, this will be the subject, God willing, of the next chapter and that will complete this investigation of the distance of the convexity of the sphere of the fixed stars.

[End Chapter 131]

IV. Commentary on Chapter 131

Levi's procedure for finding the planetary distances can be considered as a series of calculations in which the minimum distance from the center of the earth to a planet's sphere is found from the minimum distance of the planet below it according to the rules described in the Introduction. The one exception is the case of the Moon where we are given its maximum distance. Indeed, the maximum lunar, and minimum solar distances have been calculated in previous chapters and they remain fixed. However, Levi applies his method of successive approximations to the distances for Mercury and Venus involving 3 iterations before he is satisfied with the results. The procedure for adjusting parameters is hinted at in [21]–[24] and we will discuss it in the commentary *ad loc.* Note that numbers in square brackets refer to sentences in the translation of this chapter.

Ad [3]–[8]. To find the minimum distance of Mercury, we must calculate the thickness of the fluid between it and the Moon. Following assumption (a2), we find the thickness sv where ts is 60 (in Figure 2, where P_i is the Moon and P_{i+1} is Mercury) from the proportion

$$\frac{v}{360^{\circ/d}} = \frac{sv}{6;40} \tag{n1}$$

where v is the slowest daily lunar velocity to the east subtracted from 360°. Implicit in our text are 3 values for the slowest daily lunar velocity: $11;53,30^{\circ/d}$ in [4]; $11;7,25^{\circ/d}$ in [5]; and $12;7,25^{\circ/d}$ in [25]. The preferred reading is the third value, for it corresponds to the minimum entry, $0;30,18^{\circ/d}$, in Levi's table for hourly lunar velocity based on his lunar model (Goldstein 1974a, pp. 111–114, 182). Note that $12;7,25°/24 = 0;30,18,32^{\circ/d}$ which is reasonably close to the tabulated value, and much closer than that reached with the other variants.

Hence in (n1) we have

$$\frac{347;52,35^{\circ/d}}{360^{\circ/d}} = \frac{sv}{6;40} \tag{n2}$$

and so in [5]

$$sv = 6;26,30,29 \text{ (acc. } 6,26,31,45). \tag{n3}$$

We next consider the proportion

$$\frac{(tv)_1}{(tv)_2} = \frac{(tw)_1}{(tw)_2}. \tag{n4}$$

Subscript 1 refers to the measure where the radius of the concave surface of the lower planet (here, the Moon), *ts*, is taken to be 60, and subscript 2 refers to the measure where the radius of the convex surface of the higher planet (here, Mercury), *tw*, is taken to be 60. Substituting the appropriate values in (n4), we find

$$\frac{66;26,30,29}{53;20} = \frac{(tw)_1}{60} \tag{n5}$$

and so

$$(tw)_1 = 74;44,49. \tag{n6}$$

In [7] both MSS read 66;26,47,9 for $(tv)_1$ instead of the expected 66;26,30,29 as in [6]. We now use the proportion

$$\frac{TS}{(ts)_1} = \frac{TW}{(tw)_1} \tag{n7}$$

where *TS*, the greatest lunar distance is 62;48,42,36 t.r. Thus

$$\frac{62;48,42,36 \text{ t.r.}}{60} = \frac{TW}{74;44,49} \tag{n8}$$

and so the minimum distance of Mercury, r_c, in [8]

$$r_c = TW = 78;14,58 \text{ t.r.} \tag{n9}$$

It may be convenient to consider (n7) in the form

$$r_c = R_m \cdot \frac{(tw)_1}{60} \tag{n10}$$

to represent Levi's procedure for finding the minimum distance of Mercury, r_c, from the maximum distance of the Moon, ($R_m = TS$), where both are measured in earth radii.

Ad [9]–[14]. Finding the minimum distance of Venus from the minimum distance of Mercury requires several steps. First, we assume that the ratio of the true maximum and minimum distances of Mercury is the same as that found in Levi's model for that planet's motion (Mercury's motion is the subject of chapters 106, 107, and 108: unpublished). Let us consider that ratio to be D/d. Then in Figure 2 (where P_i is Mercury and P_{i+1} is Venus)

$$\frac{D}{d} = \frac{TS}{TR}. \tag{n11}$$

With the values for D/d in [9], we have

$$\frac{83;54,28}{36;\ 5,32} = \frac{TS}{78;14,58 \text{ t.r.}} \tag{n12}$$

and so the maximum distance of Mercury

$$TS = 181;54,56 \text{ t.r.} \tag{n13}$$

To find the thickness of the fluid layer between Mercury and Venus, we proceed as we did for the fluid layer between the Moon and Mercury, using the maximum retrograde daily velocity added to 360°. This velocity is given as $360;45,15^{\circ/d}$ and is not derived here (similarly, we are given the maximum retrograde velocities of the other planets).

Thus in (n1)

$$\frac{360;45,16^{\circ/d}}{360^\circ} = \frac{sv}{6;40} \tag{n14}$$

and so in [11]

$$sv = 6;40,52 \text{ (acc. } 6;40,50,17). \tag{n15}$$

According to (n4)

$$\frac{66;40,52}{53;20} = \frac{(tw)_1}{(tw)_2} \tag{n16}$$

and so in [13]

$$(tw)_1 = 75;0,56 \text{ (acc. } 75;0,58,30). \tag{n17}$$

According to (n7)

$$\frac{181;54,56 \text{ t.r.}}{60} = \frac{TW}{75;0,56} \tag{n18}$$

and so the minimum distance of Venus in [14]

$$TW = 227;26,27 \text{ t.r. (acc. } 227;26,30 \text{ t.r.).} \tag{n19}$$

It may be convenient to combine (n11) and (n18) in the form

$$TW = TS \cdot \frac{75;0,56}{60} = TR \cdot \frac{D}{d} \cdot \frac{75;0,56}{60} \tag{n20}$$

where TW is the minimum distance of Venus, TR is the minimum distance of Mercury, and D/d is the ratio of maximum to minimum distance for Mercury. If we combine (n20) with our expression for the minimum distance of Mercury, r_c, in (n10), in terms of the maximum lunar distance, R_m, the minimum distance of Venus can also be expressed in terms of the maximum lunar distance

$$r_c = R_m \cdot \frac{74;44,49}{60} \cdot \frac{D}{d} \cdot \frac{75;0,56}{60}. \tag{n21}$$

Ad [15]–[20]. Here we find the minimum solar distance from the minimum distance of Venus. According to (n11)

$$\frac{D}{d} = \frac{TS}{TR} \tag{n22}$$

and with the value for D/d in [15]

$$\frac{105;45}{15;15} = \frac{TS}{227;26,27 \text{ t.r.}}. \tag{n23}$$

Thus, the maximum distance of Venus (see Figure 2, where P_i is Venus and P_{i+1} is the Sun)

$$TS = 1562;15,27 \text{ t.r.} \tag{n24}$$

Next we apply (n1) with the appropriate value for v, as stated in [16]

$$\frac{360;44,27^{\circ/\mathrm{d}}}{360^{\circ/\mathrm{d}}} = \frac{sv}{6;40} \tag{n25}$$

and so

$$sv = 6;40,49,23. \tag{n26}$$

Substituting the appropriate values in (n4), we have

$$\frac{66;40,49,23}{53;20} = \frac{(tw)_1}{60} \tag{n27}$$

and so

$$(tw)_1 = 75;0,56. \tag{n28}$$

Applying (n7), we have

$$\frac{1562;15,27\text{ t.r.}}{60} = \frac{TW}{75;0,56} \tag{n29}$$

and so the minimum solar distance, r_s, as stated in [19]

$$r_s = TW = 1952;49,40\text{ t.r. (acc. }1953;13,37\text{ t.r.).} \tag{n30}$$

The difference between r_s and the assumed value for the minimum solar distance, 2052;4,47 t.r., mentioned in [47], is 99;15,7 t.r., mentioned in [20]. In chapter 128, the minimum solar distance is given as 2052;4,47,23 (Q 217a:16; the variant in P 245b:25, 2052;4,*48*,23, is an inferior reading). For convenience we can consider r_s in (n30) in terms of R_m, the maximum lunar distance, using (n21) and (n23)

$$r_s = R_m \cdot \frac{74;44,49}{60} \cdot \frac{D_c}{d_c} \cdot \frac{75;0,56}{60} \cdot \frac{D_v}{d_v} \cdot \frac{75;0,56}{60} \tag{n31}$$

where the subscript s refers to the Sun, m the Moon, c Mercury, and v Venus.

Ad [21]–[24]. Levi's description of his procedure for adjusting assumption (a1) is lacking in clarity, but his method can be reconstructed. On his name for this method, "heuristic reasoning", see the Introduction. His fundamental insight is to commute the ratios in (n31) such that

$$r_s = \left(R_m \cdot \frac{D_c}{d_c} \cdot \frac{D_v}{d_v}\right) \cdot \left(\frac{p_1}{60} \cdot \frac{p_2}{60} \cdot \frac{p_3}{60}\right) \tag{n32}$$

where $p_i/60$ represents the ratio of the radii of the outer and inner boundaries of an interplanetary fluid layer. In [21] we are told that

$$R_m \cdot \frac{D_c}{d_c} \cdot \frac{D_v}{d_v} = 1003;2,36\text{ t.r.} \tag{n33}$$

a result that already appears in chapter 128 (P 246a:11), and is equivalent to setting the thickness of the fluid layer at 0; in effect, this is Ptolemy's nesting principle in the *Planetary Hypotheses*. Then Levi notes that the p_i's in (n31) are all about equal to 75 and so in [23]

$$\frac{p_1}{60} \cdot \frac{p_2}{60} \cdot \frac{p_3}{60} \approx \left(\frac{75}{60}\right)^3 = \left(\frac{5}{4}\right)^3. \tag{n34}$$

We can now present a reasonable reconstruction of Levi's method for finding his initial value 6;40 in assumption (a1). In (n32) we substitute the values for the maximum solar distance and the value in (n33). Thus

$$2052;4,47 = 1003;2,36 \cdot \left(\frac{p}{60}\right)^3. \tag{n35}$$

This is equivalent to ignoring the difference between $360^{\circ/d}$ and the velocity used in (n1), for in that case the p_i's will be equal. Solving (n35), we find

$$p \approx 75. \tag{n36}$$

Now in (n4) we have

$$\frac{60 + sv}{60 - sv} = \frac{p}{60} = \frac{75}{60}. \tag{n37}$$

Thus

$$sv = 6;40 \tag{n38}$$

and this is his initial value in assumption (a1). Levi can now begin his calculation which requires an adjustment of the value for $(tv)_1$ in (n4) based on (n1), i.e., assumption (a2). Levi eliminates the discrepancy between r_s in (n30) and its assumed value in [47] by adding 0;31 to 6;40 in assumption (a1) yielding a new value of 7;11 in [24], where the method is described by the expression: heuristic reasoning. We reconstruct that method as follows: in (n32) we say that

$$2052;4,47 = 1003;2,36 \cdot \left(\frac{p}{60}\right)^3$$

$$= 1003;2,36 \cdot \left(\frac{75 + x}{60}\right)^3. \quad \text{(n39)}$$

If we use the following approximation

$$(75 + x)^3 \approx 75^3 + 3 \cdot 72^2 x \quad \text{(n40)}$$

and substitute it in (n39), we find that

$$x \approx 1;12 \quad \text{(n41)}$$

and so

$$P \approx 76;12. \quad \text{(n42)}$$

We now substitute this value of p in (n37), so that

$$\frac{60 + sv}{60 - sv} = \frac{76;12}{60} \quad \text{(n43)}$$

and so

$$sv \approx 7;11 \text{ (acc. } 7;8). \quad \text{(n44)}$$

Ad [25]–[29]. With the new value, 7;11 in assumption (a1), Levi re-computes the minimum distance of Mercury from the maximum lunar distance, but some steps are omitted in the text. To find the thickness of the fluid layer between the Moon and Mercury, we apply (n1) with the data in [25]

$$\frac{347;52,35^{o/d}}{360^{o/d}} = \frac{sv}{7;11} \quad \text{(n45)}$$

and so

$$sv = 6;56,29,7. \quad \text{(n46)}$$

Then, according to (n4),

$$\frac{66;56,29,7}{52;49} = \frac{(tw)_1}{60} \tag{n47}$$

and so

$$(tw)_1 = 76;2,45. \tag{n48}$$

Finally, according to (n7),

$$\frac{62;48,42,36 \text{ t.r.}}{60} = \frac{TW}{76;2,45} \tag{n49}$$

and so r_c, the minimum distance of Mercury,

$$r_c = TW = 79;36,33 \text{ t.r.} \tag{n50}$$

Ad [30]–[50]. To find the minimum distance of Venus from the minimum distance of Mercury, we must first find the maximum distance of Mercury. According to (n11)

$$\frac{83;54,28}{36;\ 5,32} = \frac{TS}{79;36,33 \text{ t.r.}} \tag{n51}$$

and so R_c, the maximum distance of Mercury,

$$R_c = TS = 185;4,36 \text{ t.r.} \tag{n52}$$

According to (n1),

$$\frac{360;45,16^{\circ/\text{d}}}{360^{\circ/\text{d}}} = \frac{sv}{7;11} \tag{n53}$$

and so

$$sv = 7;11,54,12. \tag{n54}$$

Then, according to (n4)

$$\frac{67;11,54,12}{52;49} = \frac{(tw)_1}{60} \tag{n55}$$

and so

$$(tw)_1 = 76;20,20 \text{ (acc. } 76;20,16). \tag{n56}$$

Finally, according to (n7),

$$\frac{185;4,36 \text{ t.r.}}{60} = \frac{TW}{76;20,20} \tag{n57}$$

and so r_v, the minimum distance of Venus,

$$r_v = TW = 235;28,33 \text{ t.r.} \tag{n58}$$

Ad [36]–[41]. To find the minimum solar distance from the minimum distance of Venus, we must first find the maximum distance of Venus. According to (n11)

$$\frac{104;45}{15;15} = \frac{TS}{235;28,33 \text{ t.r.}} \tag{n59}$$

and so R_V, the maximum distance of Venus,

$$R_v = TS = 1617;26,56 \text{ t.r.} \tag{n60}$$

According to (n1)

$$\frac{360;44,24^{\circ/d}}{360^{\circ/d}} = \frac{sv}{7;11} \tag{n61}$$

and so

$$sv = 7;11,53. \tag{n62}$$

Then, according to (n4)

$$\frac{67;11,53}{52;49} = \frac{(tw)_1}{60} \tag{n63}$$

and so

$$(tw)_1 = 76;20,14. \tag{n64}$$

Finally, according to (n7)

$$\frac{1617;26,56 \text{ t.r.}}{60} = \frac{TW}{76;20,14} \tag{n65}$$

and so r_s, the minimum solar distance,

$$r_s = TW = 2057;46,54 \text{ t.r. (acc. } 2057;51,34 \text{ t.r.)} \tag{n66}$$

and this is 5;42,47 (acc. 5;42,7) greater than the previously determined minimum solar distance of 2052;4,47 mentioned in [47].

Ad [42]–[47]. Because of the discrepancy in the solar distance noted in [41], Levi applied his adjustment procedure again (as in [21]–[24]) and tells us to diminish the value in assumption (a1) from 7;11 to 7;9,20. Then, for the final time, he recomputes the minimum solar distance leaving out a few steps. Following the same procedure as before with the new data, we find in (n4)

$$(tv)_1 = 66;54,52,19 \tag{n67}$$

and

$$(tw)_1 = 75;78,31,15. \tag{n68}$$

(This step is not in the text.) Then, according to (n7)

$$\frac{62;48,42,36 \text{ t.r.}}{60} = \frac{TW}{75;58,31,15} \tag{n69}$$

and so r_c, the minimum distance of Mercury,

$$r_c = 79;32 \text{ t.r.} \tag{n70}$$

as in [43]. To find the minimum distance of Venus, we apply (n11) with the ratio for Mercury's distances

$$\frac{53;54,28}{36;\ 5,32} = \frac{TS}{79;32 \text{ t.r.}} \tag{n71}$$

and so R_c, the maximum distance of Mercury

$$R_c = TS = 184;54 \text{ t.r.} \tag{n72}$$

Then, according to (n4),

$$\frac{67;10,14}{52;50,40} = \frac{(tw)}{60} \tag{n73}$$

and so

$$(tw)_1 = 76;15,57.$$

Applying (n11)

$$\frac{184;54 \text{ t.r.}}{60} = \frac{TW}{76;15,57} \tag{n74}$$

and so r_v, the minimum distance of Venus

$$r_v = TW = 235;2 \text{ t.r.} \tag{n75}$$

which is in [45]. Next, according to (n11)

$$\frac{104;45}{15;15} = \frac{TS}{235;2 \text{ t.r.}} \tag{n76}$$

and so R_c, the maximum distance of Venus

$$R_c = TS = 1614;23 \text{ t.r. (acc. } 1614;25 \text{ t.r.)} \tag{n77}$$

which is in [46]. We now seek the thickness of the fluid layer between Venus and the Sun. According to (n1)

$$\frac{360;44,27^{\circ/d}}{360^{\circ/d}} = \frac{sv}{7;9,20} \tag{n78}$$

and so

$$sv = 7;10,13.$$

Then, according to (n4)

$$\frac{67;10,13}{52;50,40} = \frac{(tw)_1}{60} \tag{n79}$$

and so

$$(tw)_1 = 76;15,56. \tag{n80}$$

Finally, according to (n7)

$$\frac{1614;23 \text{ t.r.}}{60} = \frac{TW}{76;15,56} \tag{n81}$$

and so r_s the minimum solar distance,

$$r_s = TW = 2052;4,47 \text{ t.r. (acc. } 2052;2 \text{ t.r.).} \tag{n82}$$

Ad [48]–[54]. To find the minimum distance of Mars from the minimum solar distance, we first determine the maximum solar distance, which, we are told in [49], is 2229;29 t.r. The eccentricity that corresponds to the stated minimum and maximum solar distances is about 2;29 rather than Levi's final value of about 2;23 (Goldstein 1974a, p.

94), his preliminary value of 2;14, or Ptolemy's value of 2;30. The eccentricity, e, may be derived from the following expression

$$\frac{D}{d} = \frac{60 + e}{60 - e} = \frac{2229;29 \text{ t.r.}}{2052;4,47 \text{ t.r.}} \tag{n83}$$

and solving for e. To find the thickness of the fluid layer between the Sun and Mars, we apply (n1)

$$\frac{359;3,12^{\circ/\text{d}}}{360^{\circ/\text{d}}} = \frac{sv}{7;9,20} \tag{n84}$$

and so

$$sv = 7;8,12 \tag{n85}$$

The minimum solar velocity to the east, $0;56,48^{\circ/\text{d}}$, corresponds to $0;2,22^{\circ/\text{h}}$, a value consistent with Levi's final value for the solar eccentricity of about 2;23 (Goldstein 1974a, pp. 94, 110). Next, we apply (n4)

$$\frac{67;\ 8,12}{52;50,40} = \frac{(tw)_1}{60} \tag{n86}$$

and so

$$(tw)_1 = 76;14,24,26 \text{ (acc. } 76;13,39). \tag{n87}$$

Finally, according to (n7)

$$\frac{2229;29 \text{ t.r.}}{60} = \frac{TW}{76;14,24,26} \tag{n88}$$

and so r_r, the minimum distance of Mars,

$$r_r = TW = 2832;56 \text{ t.r.}$$

Ad [55]–[60]. The method for finding the minimum distance of Jupiter from the minimum distance of Mars has been discussed in the Introduction.

Ad [61]–[66]. To find the minimum distance of Saturn from the minimum distance of Jupiter, we first find the maximum distance of Jupiter. Applying (n11), we have

$$\frac{73;28,36}{46;31,24} = \frac{TS}{24038;27 \text{ t.r.}} \tag{n89}$$

and so R_j, the maximum distance of Jupiter

$$R_j = TS = 37965;8 \text{ t.r.} \tag{n90}$$

Applying (n1)

$$\frac{360;8,11^{\text{o/d}}}{360^{\text{o/d}}} = \frac{sv}{7;9,20} \tag{n91}$$

and so

$$sv = 7;9,28 \text{ (acc. } 7;9,30). \tag{n92}$$

According to (n4)

$$\frac{67;\ 9,28}{52;50,40} = \frac{(tw)_1}{60} \tag{n93}$$

and so

$$(tw)_1 = 76;15,5. \tag{n94}$$

Finally, according to (n7),

$$\frac{37965 \text{ t.r.}}{60} = \frac{TW}{76;15,5} \tag{n95}$$

and so r_t, the minimum distance of Saturn,

$$r_t = TW = 48248;15 \text{ t.r.} \tag{n96}$$

Ad [67]–[72]. To find the minimum distance of the sphere of the fixed stars from the minimum distance of Saturn, we first find the maximum distance of Saturn. Applying (n11)

$$\frac{69;18,52}{50;41,8} = \frac{TS}{48248;15 \text{ t.r.}} \tag{n97}$$

and so R_t, the maximum distance of Saturn,

$$R_t = TS = 65981;21 \text{ t.r.} \tag{n98}$$

Next we apply (n1)

$$\frac{360;4,38^{\circ/\mathrm{d}}}{360^{\circ/\mathrm{d}}} = \frac{sv}{7;9,20} \tag{n99}$$

and so

$$sv = 7;9,25. \tag{n100}$$

According to (n4)

$$\frac{67;\ 9,25}{52;50,40} = \frac{(tw)_1}{60} \tag{n101}$$

and so

$$(tw)_1 = 76;15. \tag{n102}$$

Finally, according to (n7)

$$\frac{65981;21 \text{ t.r.}}{60} = \frac{TW}{76;15} \tag{n103}$$

and so R_f, the minimum distance to the sphere of the fixed stars,

$$R_f = TW = 83851;18 \text{ t.r.} \tag{n104}$$

Ad [73]-[75]. The thickness of the sphere of the fixed stars is determined by the diameter of the largest star and that value is computed in chapter 132. There the calculation of stellar diameters depends on the method in Ptolemy's *Planetary Hypotheses* using the apparent diameters indicated in that text together with the distances derived by Levi here. In chapter 134 Levi takes the Sun to be below Mercury and Venus, and begins with an initial value of 56;29,24 in assumption (a1) with the result that the distance to the fixed stars is more than 157×10^{12} t.r. To explain the parameter, 56;29,24, we may recall the derivation in the comments ad [21]–[24]. In Figure 2, P_i is now the Moon and P_{i+1} is the Sun. If we consider the motion at the maximum distance of the Moon to be $360^{o/d}$, then according to (n7)

$$\frac{62;48,42,36 \text{ t.r.}}{60} = \frac{2052;4,47 \text{ t.r.}}{(tw)_1} \tag{n105}$$

and so

$$(tw)_1 \approx 1960. \tag{n106}$$

If we consider the ratio of the radii of the outer and inner boundaries of the fluid layer to be $(tw)_1/60$, then, according to (n4)

$$\frac{60 + sv}{60 - sv} = \frac{1960}{60}. \tag{n107}$$

It follows that

$$sv \approx 56;26$$

which is very nearly the value in chapter 134 cited above. Moreover, from the discussion in the comments ad [21]–[24], it is clear that this large value for the distance to the sphere of fixed stars, R, can be approximated by the following expression

$$R \approx R_s \cdot \frac{D_c}{d_c} \cdot \frac{D_v}{d_v} \cdot \frac{D_r}{d_r} \cdot \frac{D_j}{d_j} \cdot \frac{D_t}{d_t} \cdot \left(\frac{p}{60}\right)^6 \tag{n108}$$

where R_s is the maximum solar distance; D/d is the ratio in the planetary model for maximum to minimum distance and the subscripts refer in turn to Mercury, Venus, Mars, Jupiter, and Saturn; and $p/60$ represents the ratio of the radii of the outer and inner boundaries of an interplanetary fluid layer. Thus, in (n108), using rounded values from Table 1,

$$R \approx 2229 \cdot \frac{89}{36} \cdot \frac{105}{15} \cdot \frac{104}{16} \cdot \frac{73}{47} \cdot \frac{69}{51} \cdot \left(\frac{2000}{60}\right)^6 \qquad \text{(n109)}$$

$$\approx 500 \cdot 10^{12} \text{ t.r.}$$

This distance is of the same order of magnitude as Levi's value which is based on a more refined procedure than the one used here.

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