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ON THE BABYLONIAN DISCOVERY OF THE PERIODS OF LUNAR MOTION

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A most impressive achievement in Babylonian astronomy was the development of lunar theories that yielded very good results enabling the Babylonians to compute lunar eclipses with great success.[1] Fundamental to these theories was the discovery of the periods for lunar motion (the sidereal, synodic, draconitic, and anomalistic months), and these periods had been accurately determined by about 500 B.C. It is by no means obvious how the Babylonians were able to find such accurate parameters, given that their observations were relatively crude, and they do not offer any indication of the way that these parameters were derived from the observations at their disposal. The most important description of Babylonian lunar theories was given by Neugebauer in 1955,[2] but he did not address the question, How were the periods derived from observations? Significant progress concerning this question has been made possible by the publication of the Babylonian Astronomical Diaries that report almost daily observations of the heavens from the eighth to the first century B.C. (although many of the tablets in this series have not been preserved, especially at the beginning of it).[3] Moreover, Britton and Brack-Bernsen (working separately) have recently proposed reconstructions of the methods used by the Babylonians to derive these parameters, and I will follow most of what they have suggested, adding some arguments for the way the Babylonians may have depended on simple observations over a relatively short time period.[4] To be sure, in the absence of specific derivations by the Babylonians, one can hardly be certain of any reconstruction, and I do not claim that the Babylonians followed the route described here. Rather, it is my goal to indicate that reaching initial values for those parameters required neither precise observations nor a very long time interval, and the same may also hold true for the very accurate parameters that are embedded in their theories. I am well aware that others, including some more qualified than I, interpret the historical data differently, and I encourage them to use this journal as a forum for discussing the origins of mathematical astronomy.

We begin with the draconitic, synodic, and sidereal months, and then turn to the more difficult case of the anomalistic month. To find initial values for the draconitic, synodic, and sidereal months, the Babylonians could have proceeded from the following data alone:

1. There are 38 eclipse possibilities in 223 synodic months (about 18 years).
2. 235 synodic months = 19 years.
3. 1 Saros = 223 synodic months ≈ 6585;20d.

These assumptions are all well attested in early Babylonian documents, but they

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need some explanation. As for the first assumption, Aaboe *et al.* indicate that "by 'eclipse possibility' we mean a syzygy [conjunction or oppostion] at which the sun is within half a month's progress in elongation from a lunar node. At such times ... occur lunar eclipse possibilities at opposition".[5] In other words, a lunar eclipse is considered possible when at opposition the sun lies in one nodal zone and the moon in the other, i.e., where the lunar latitude is small. Some eclipses will not be seen, for example, because they take place during the daytime. Britton offered a reconstruction of the way the Babylonians could get this relationship from observable lunar eclipses recorded over a few decades by only the regnal year and month in the Babylonian calendar, provided that they kept track of the number of months that had elapsed, and this reconstruction is most persuasive. The insight that the Babylonians had at an early date is that observed lunar eclipses are separated either by 6 synodic months, or by one less than multiples of 6 synodic months (reflecting an occasional 5-month interval).[6] The Saros canon and related extant texts provide evidence that the Babylonians were aware of this relationship, for they list dates of lunar eclipses starting from –490 in columns of 38 entries covering 223 synodic months such that entries in adjacent columns are separated by 223 synodic months. The beginning of the relevant tablet is broken; in its original unbroken state, it probably listed eclipses as far back as –526.[7] The second assumption, that 235 synodic months is equal to 19 years, implies there are 19 complete returns in longitude for the sun and 254 (= 19 + 235) complete returns in longitude for the moon.[8] The third assumption is also attested in early texts.[9] Note that 3 Saroi correspond very nearly to an integer number of days; hence, this quantity, as the others, depends only on counting without appealing to precise measurement.

To find a value for the draconitic month, we may proceed as follows. By Assumption 1, the mean interval between eclipse possibilities is:

$$223/38 = 5;52,6,18,\ldots\, m,$$

where m is a synodic month. Since the distance between successive nodes is 180°, the lunar progress per synodic month with respect to the node, in addition to a complete revolution, is:

$$180/5;52,6,18,\ldots = 30;40,21,\ldots{}^\circ/m.$$

Hence the total progress of the moon in 223 synodic months is:

$$223 \cdot 30;40,21,\ldots = 6840^\circ$$

and this corresponds to 19 revolutions, for

$$6840/360 = 19 \text{ revolutions.}$$

Finally, taking into account a complete revolution in each of the 223 months,

$$19 + 223 = 242 \text{ revolutions (or draconitic months).}$$

We can combine the preceding steps to indicate that 19 revolutions is a precise value, rather than an approximation:

$$(38 \cdot 180 \cdot 223)/(223 \cdot 360) = 19 \text{ revolutions.}$$

It follows that the length of the draconitic month is:

$$6585;20/242 = 27;12,43,38,... \text{ d.}$$

This result is very close to the modern value for the draconitic month:[10]

$$1 \text{ dra. month} = 27\text{d } 5;5,35,50\text{h} = 27;12,43,59,35\text{d.}$$

We next turn to the sidereal month. The first step is to find the length of a synodic month from the number of days in 223 synodic months (Assumption 3, above):

$$6585;20\text{d}/223m = 29;31,50,18,... \text{ d}/m.$$

Then we can determine the length of 235 months (= 19 years):

$$29;31,50,18 \cdot 235 \approx 6939;42\text{d},$$

or, alternatively, using a rounded value for the mean synodic month:

$$29;31,50 \cdot 235 \approx 6939;41\text{d}.$$

Now we can find the length in days of the sidereal month, where 254 (= 235 + 19) sidereal months is equal to 19 years (based on Assumption 2, above):

$$6939;41/254 \approx 27;19,17,43\text{d/sid. month.}$$

It follows that the mean motion in longitude is:

$$360/27;19,17,43 = 13;10,35°/\text{d},$$

and this is the standard Babylonian value for the daily mean motion in longitude. On the other hand, if one takes 19 years to equal 6939;42d, one gets a sidereal month of 27;19,17,57d, and a mean motion of 13;10,34,54°/d ≈ 13;10,35°/d. Again these results are very close to the modern value for the sidereal month:[11]

$$1 \text{ sid. month} = 27\text{d } 7;43,11,30\text{h} = 27;19,17,58,45\text{d.}$$

Unlike other astronomical periodicities known in Antiquity, lunar anomaly is difficult to observe (its period is the period of lunar velocity) and, until recently, there was no phenomenon that seemed to allow lunar velocity to be derived from the kinds of observations available to the ancients. Indeed, it is not obvious how the phenomenon of lunar anomaly was first noticed. This state of affairs changed in 1993 when Brack-Bernsen demonstrated, using modern data, that a set of related observed quantities, regularly tracked by the Babylonians, can yield the period of lunar velocity without the interference of other factors.[12] From a modern point of view, lunar velocity is much more complicated in general than it is at syzygy, and that makes it difficult to derive the period of velocity from daily increments in lunar longitude throughout the synodic month. In fact, the Babylonians did not record daily observations of the moon, and I find it unlikely that the daily progress of the moon in longitude played any role in the derivation of the period of lunar anomaly.[13] Conjunctions of the sun and the moon take place during the period of lunar invisibility except on the rare occasions when there is a solar eclipse, and so conjunctions cannot be used to derive lunar velocity. On the other hand, the Babylonians in the second millennium had already shown an interest in lunar eclipses (that take place at oppositions) as we learn from the omen series, *Enūma Anu Enlil*,[14] and the discovery of the period of lunar anomaly almost certainly came about from their interest in the phenomena surrounding oppositions.

The period of lunar anomaly underlies what is called in the modern literature Column Φ in Babylonian lunar System A, and the basic arithmetic structure of this column of numbers has been known for some time.[15] It is a linear zigzag function with entries to six sexagesimal places, and a period of 6247 synodic months (= 6695 anomalistic months), or about 505 years. Moreover, it is in phase with column F in lunar System A that lists the lunar velocities at the beginning and the middle of synodic months. Neugebauer used the entries in Column Φ as an indexing function, for there is no repeat of a value in that column within 505 years; the entries in Column Φ all belong to the same series tabulated at intervals of half a synodic month, but they are generally separated into Φ_1 for conjunctions and Φ_2 for oppositions. We are interested in the way the Babylonians may have first computed a value for the anomalistic month based on observations over a relatively short interval of time. A particularly accurate value for the anomalistic month (27;33,16,30d) appears in Babylonian lunar System A, and a particularly accurate period relation for lunar anomaly appears in System B (but not explicitly in System A): 251 synodic months correspond to 269 anomalistic months. Britton has presented a persuasive argument for the claim that the period of 6247 synodic months of Column Φ in System A was derived arithmetically from the Saros period of 223 synodic months with the aim of approximating the period of 251 synodic months which, by modern standards, is more accurate.[16] In Britton's derivation, it was assumed that the periods of 223 synodic months and 251 synodic months were known to the author(s) of System A and, if so, no additional observations were needed to produce the period of 6247 months that underlies Column Φ (see Appendix 1). But, as far as I am aware, the derivation of the period of 251 months has not been satisfactorily explained, i.e., there is no plausible hypothesis for the way this period might have been derived from simple observations over a relatively short time interval. Here we will seek to provide a way for this period to be derived arithmetically from shorter periods without recourse to precise measurements. It will be argued that the length of the synodic month in System B (29;31,50,8,20d) was probably known to the author(s) of System A as well as the period of 251 synodic months, although neither one is explicitly mentioned in any available text of System A.

To derive the period relation, 251 synodic months = 269 anomalistic months, we need some preliminary information. For this purpose we appeal to two different periods of lunar motion (with their approximate lengths) that we have already discussed: the synodic month (29;31,50 days), and the sidereal month (27;19 days).

The most important step in this reconstruction of the discovery of lunar anomaly was taken by Brack-Bernsen who introduced a function called Σ that is defined as the sum of *shu*, *na*, *me*, *ge*, four quantities measured by the Babylonians in time-degrees just before and just after opposition.[17] The first, *shu*, is defined as the time between moonset and sunrise when the moon sets for the last time before sunrise; the second, *na*, is the time between sunrise and moonset when the moon sets for the first time after sunrise; the third, *me*, is the time between moonrise and sunset when the moon rises for the last time before sunset; and the fourth, *ge*, is the time between sunset and moonrise when the moon rises for the first time after sunset.[18] These quantities, known as the 'lunar fours', were, in principle, observed each month. Surprisingly, this function, Σ, has the period of lunar velocity and it is in phase with column Φ.[19]

Let us grant that the Babylonians had a series of values for the lunar fours, and that they were excerpted from the Astronomical Diaries. Moreover, let us assume that the Babylonians also added the values for the lunar fours (producing a set of Σs) corresponding to a sequence of oppositions.[20] The period of 14 synodic months stands out as the shortest period for Σ, but it is not obvious how many returns in lunar velocity correspond to it, particularly since the data only relate to oppositions. Something is needed to get started, and I think it reasonable that initially the anomalistic period was assumed to be roughly the same as the sidereal period. In the case of the sun, it is in fact true that velocity depends on longitude alone (and so it is in the Babylonian theories for solar motion), and it is plausible to consider this for the moon as well. On the other hand, the Babylonians had probably already determined that the draconitic month is slightly different from the sidereal month. So let us first assume that 14 synodic months is approximately equal to 15 sidereal months. How then would one recognize that the sidereal month is different from the anomalistic month? After three periods of 14 months or 42 months, the difference between them is surely noticeable:

$$42m = 42 \cdot 29;31,50\text{d} = 1240;17\text{d}$$

and, where the mean motion in longitude of the moon is 13;10,35°/d,

$$1240;17\text{d} \cdot 13;10,35°/\text{d} = 45 \text{ rotations} + 142°,$$

i.e., after 42 synodic months there is not a return in lunar longitude. But, if it is still maintained that 14 synodic months correspond to 15 returns in Σ, then 42 synodic months correspond to 45 returns in Σ or, in other words, to 45 returns in velocity. Hence

$$1240;17\text{d}/45 \approx 27;33\text{d},$$

which is about the anomalistic period. Thus it would be noticed very quickly that the anomalistic period is greater than the sidereal period of about 27;19 days.

Further, one can improve this result by using the Saros cycle. Although this cycle was probably first invoked as a good period for lunar eclipses (as noted above), it is also a good period for lunar anomaly. It has been suggested that, by comparing the values for Σ at intervals of 223 months, one might notice that the complicated fine structure of the Σ curve repeats, very nearly, after 223 months.[21] But, more simply, if the length of an anomalistic month had already been estimated to be about 27;33d, it follows that

$$6585;20/27;33 \approx 239 \text{ anomalistic months.}$$

We can then compute the length of an anomalistic month from the number of days in 223 synodic months:

$$6585;20/239 \approx 27;33,13,18\text{d.}$$

This value for the anomalistic month is an improvement over the first value we found for this parameter, but the Babylonians were able to do even better.

Brack-Bernsen showed, on the basis of the function, Σ, that usually the anomaly returns in 14 synodic months, but sometimes there is a period of 1 less than a multiple of 14 synodic months.[22] One may then ask, How often are there units of 13*m* in periods of anomaly? In 223*m* there are 15 units of 14*m* and only 1 unit of 13*m*, for $15 \cdot 14 + 13 = 223$, i.e., the ratio of synodic to anomalistic month is greater than 15 to 14. After how many more units of 14*m* is there a unit of 13*m*? If the Babylonians knew that 223*m* was a little shorter than 239 anomalistic months (*ma*), then

$$\frac{14}{15} > \frac{ma}{m} > \frac{223}{239}.$$

Now, a theorem known in Antiquity[23] states that if

$$\frac{a}{b} > \frac{c}{d}$$

then

$$\frac{a}{b} > \frac{a+c}{b+d} > \frac{c}{d}.$$

Applying this theorem, we can arrive at the refined Babylonian period relation for the anomalistic month, used in System B:

$$\frac{14}{15} = \frac{28}{30} > \frac{223+28}{239+30} > \frac{223}{239}$$

or

$$\frac{14}{15} > \frac{251}{269} > \frac{223}{239}.$$

Thus, the period relation, $251m = 269$ anomalistic months, can be determined without requiring any additional observations. Moreover, with the ratio, 251 to 269, and a Babylonian value for the length of the mean synodic month (see Appendix 2), one can easily get 27;33,16,30d as the length of the anomalistic month in System A without additional observations,[24] for:

$$(251/269) \cdot 29;31,50,8,20\text{d} = 27;33,16,27\text{d} \approx 27;33,16,30\text{d}.$$

Note again that the modern value for this parameter is 27;33,16,30,48,...d (for –500), and the Babylonian value in System A is very close to it.[25]

The evidence for this refined value of the anomalistic month comes from the so-called Saros text that belongs to System A where 1 anomalistic month is set equal to 2 · 82;39,49,30H (where 6H = 1d), or 1 anomalistic month = 27;33,16,30d. Moreover, in the same text the length of the synodic month is given as 29;31,50,19,11,4,56d.[26] As Neugebauer noted, in System A the ratio of the synodic to the anomalistic month is 6695 to 6247. Hence:

$$(6247/6695) \cdot 29;31,50,19,11,4,56 = 27;33,16,29,59,58,\ldots\text{d} \approx 27;33,16,30\text{d}$$

or, alternatively:

$$(6695/6247) \cdot 27;33,16,30 = 29;31,50,19,11,\underline{6},\underline{45},\ldots\text{d}.$$

How should we interpret the two sets of values for the period relations of 251 and 6247 synodic months, respectively, both of which are coherent? They have in common the same value for the anomalistic month, but the three other numbers differ. With Britton, I will assume that the relationship of 251 synodic months was known prior to the relationship of 6247 synodic months, and this leads to the question of whether the length of the anomalistic month was derived from that of the synodic month in the period of 251 synodic months, or vice versa. It is difficult to decide because there is no simple way to account for the accuracy of either parameter. Nevertheless, in the absence of a viable way to get this length of the anomalistic month directly from observations or from some combination of simpler parameters,[27] I think it better to assume that the length of the synodic month was used to derive the length of the anomalistic month, using the period of 251 synodic months. According to this scenario, when the period relation was changed from 251 months to 6247 months, one of the month lengths had to be changed as well. And the decision by the Babylonians was to keep the length of the anomalistic month, while changing the synodic month to accord with it. So, *ex hypothesi*, the length of the synodic month, 29;31,50,19,11,4,56d, should have been derived from the length of the anomalistic month, 27;33,16,30d, using the period of 6247 synodic months, and, indeed, this is the procedure described in the Saros text. It is relatively unusual to find a computational error in a Babylonian astronomical text, but in this case there is no doubt: the text reads ...4,56d, whereas Neugebauer computed correctly that it should be ...6,45d.[28] To be sure this error is of no consequence, for the Babylonians carried out the computation to too many places. Moreover, while this value for the synodic month is worse than the value of 29;31,50,8,20d (from a modern point of view), it does conform to the Saros, for:

$$223 \cdot 29;31,50,19,11,4,56 = 6585;20,1,\ldots\text{d},$$

which is very close to the standard value of 6585;20d.

Critical to the previous argument are values for the ratio of *ma* to *m*. We may now consider the plausibility of this result, compared to similar computed values. Let us consider seven possibilities for this ratio (see below). We first compute a value for the anomalistic month based on a synodic month of 29;31,50d. Then, with a synodic month of 29;31,50,8,20d, and an anomalistic month of 27;33,16,30d (both of which are virtually indistinguishable from 'reality'), we compute the difference between the number of days in an integer number of synodic months and the number of days in the corresponding integer number of anomalistic months. Our goal is to minimize this difference.

1. Let

$$\frac{ma}{m} = \frac{14}{15} = 0;56.$$

Then ma = 27;33,42,40d, where $m \approx$ 29;31,50 days. But, where m = 29;31,50,8,20d and ma = 27;33,16,30d, 14m = 413;26d and 15ma = 413;19d, i.e., the difference is 0;7d.

2. Let

$$\frac{ma}{m} = \frac{223}{239} = \frac{14 \cdot 15 + 13}{15 \cdot 15 + 14} = 0;55,58,59.$$

Then ma = 27;33,12,38d, where $m \approx$ 29;31,50 days. But, where m = 29;31,50,8,20d and ma = 27;33,16,30d, 223m = 6585;19d and 239ma = 6585;33d, i.e., the difference is –0;14d.

3. Let

$$\frac{ma}{m} = \frac{223 + 14}{239 + 15} = \frac{14 \cdot 16 + 13}{15 \cdot 16 + 14} = 0;55,59,3.$$

Then ma = 27;33,14,36d, where $m \approx$ 29;31,50 days. But, where m = 29;31,50,8,20d and ma = 27;33,16,30d, 237m = 6998;45d and 254ma = 6998;52d, i.e., the difference is –0;7d.

4. Let

$$\frac{ma}{m} = \frac{223 + 2 \cdot 14}{239 + 2 \cdot 15} = \frac{14 \cdot 17 + 13}{15 \cdot 17 + 14} = \frac{251}{269} = 0;55,59,6.$$

Then ma = 27;33,16,5d, where $m \approx$ 29;31,50 days. But, where m = 29;31,50,8,20d and ma = 27;33,16,30d, 251m = 7412;11d and 269ma = 7412;11d, i.e., the difference is 0;0d.

5. Let

$$\frac{ma}{m} = \frac{14 \cdot 16 + 2 \cdot 13}{15 \cdot 16 + 2 \cdot 14} = \frac{250}{268} = \frac{125}{134} = 0;55,58,13.$$

Then ma = 27;32,50d, where $m \approx$ 29;31,50 days. But, where m = 29;31,50,8,20d and ma = 27;33,16,30d, 125m = 3691;19d and 134ma = 3692;18d, i.e., the difference is –0;59d.

6. Let

$$\frac{ma}{m} = \frac{14 \cdot 18 + 13}{15 \cdot 18 + 14} = \frac{265}{264} = 0;55,59,9.$$

Then ma = 27;33,17,42d, where $m \approx$ 29;31,50 days. But, where m = 29;31,50,8,20d and ma = 27;33,16,30d, 265m = 7825;36d and 284ma = 7825;30d, i.e., the difference is 0;6d.

7. Let

$$\frac{ma}{m} = \frac{14 \cdot 17 + 2 \cdot 13}{15 \cdot 17 + 2 \cdot 14} = \frac{264}{283} = 0;55,58,18.$$

Then ma = 27;32,52d, where $m \approx$ 29;31,50 days. But, where m = 29;31,50,8,20d and ma = 27;33,16,30d, 264m = 7796;4d, but 283ma = 7797;57d, i.e., the difference is –1;53d.

From the differences obtained using the refined Babylonian values for the synodic and anomalistic months, clearly Item 4 is best, but Items 3 and 6 are very good. It seems unlikely that the relative quality of these three period relations could be decided observationally by the Babylonians. But they could conclude that there was no reason to insert a second unit of 13 months within a time interval of 265 synodic months.

In sum, despite the absence of a Babylonian story telling us how they arrived at fundamental astronomical parameters from their observational data, one can reconstruct simple methods that do not require precise observations or the appeal to long time periods of observational records. However, it is not possible, without further indications by the Babylonians themselves, to say why these values were preferred over similar values that are very close to, and observationally indistinguishable, from them. But this may not be the right way to consider the question, for in some instances the Babylonian astronomers accepted multiple values for the same parameter that are very close to each other, as is the case for the period of lunar anomaly. As has long been known, the values for the lunar period relations in Ptolemy's *Almagest* are clearly indebted to the Babylonians[29] and, once these values were in hand, it was much easier to confirm them, or redetermine them in other ways. It is fair to say that, at least as far as lunar theory is concerned, science began in Babylon.

APPENDIX 1: THE PERIOD OF COLUMN Φ

Britton argued that a quantity, ε, was found by equating the Saros and the anomalistic period relations:[30]

$$\frac{223 + \varepsilon}{239 + \varepsilon} = \frac{251}{269}.$$

It follows that $\varepsilon = 1/9$ (exactly):

$$\frac{223 + 1/9}{239 + 1/9} = \frac{251}{269}$$

for

$$\varepsilon = (251 \cdot 239 - 223 \cdot 269)/(269 - 251) = 2/18 = 1/9.$$

As Britton noted, a value of ε equal to 1/9 corresponds to a zigzag function with only 251 discrete values, and this is too few for the purpose of indexing successive oppositions over a long period of time. So a value of ε near 1/9 was sought (to preserve the period relation, very nearly) from the following choices that are just below and just above 1/9, in the order of increasing numerators: 2/17 and 2/19; 3/26 and 3/28; 4/35 and 4/37; etc. And ε = 3/28 (or 1/9;20) was chosen by the Babylonians, in Britton's words, "perhaps because ... it contains the smallest irregular number as a factor, or possibly because 14 turns up elsewhere in the theory of lunar anomaly" (where 'regular' numbers only have factors 2, 3, and 5, while all other numbers are considered 'irregular'). The two periods are very nearly equal, for

$$251/269 = 0;55,59,6,28,6$$

and

$$6247/6695 = 0;55,59,6,13,42,$$

where 6247 = 28 · 223 + 3, and 6695 = 28 · 239 + 3.

APPENDIX 2: THE SYNODIC MONTH

It is not easy to reconstruct how the Babylonians obtained the value for the synodic month in System B, 29;31,50,8,20 days, that is very close to the modern value of 29;31,50,8,40 days. On the other hand, it is not very hard to imagine that they found a value of 29;31,50d, which is equivalent to saying that 12 synodic months = 354;22d, a parameter that served in the Middle Ages as the basis of the Hijra calendar. But the difference between it and 29;31,50,8,20d accumulates to only about 4 hours in 500 years; in 60 years, the difference accumulates to only about 0;40h. For this reason, I doubt that the precise Babylonian value was based on observations directly. Admittedly, I have not found a simple way of combining known cycles to get this basic parameter, but I found a computation that comes close to yielding it.

As a lower bound, take 10631d = 360*m* (*m* = 29;31,50d), and as an upper bound, the Saros: 6585;20d = 223*m* (*m* = 29;31,50,18,50,... d). As indicated in *Almagest* (iv.2), 3 Saroi are very close to an integer number of days. Then, using the theorem presented above,

$$\frac{10631}{360} < \frac{10631 + 6585;20}{360 + 223} < \frac{6585;20}{223}$$

and

$$\frac{17216;20}{583} = 29;31,50,7,12,...\text{d}$$

This value is very close to the Babylonian parameter, and the difference between them accumulates to only a little more than 0;45h in 4267 synodic months (about 500 years).[31]

On the other hand, Britton has argued that the refined Babylonian value for the synodic month can be derived, very nearly, from the sidereal and anomalistic months. First, Britton derived this Babylonian parameter from the 19-year cycle of 235 synodic months and the Babylonian value for the daily mean motion in longitude:[32]

$$(254/235) \cdot (360/13;10,35) = 29;31,50,4,54,...\ \text{d},$$

and the implied value for the sidereal month is 27;19,17,45,... d (= 360°/13;10,35°/d). More recently, Britton derived the synodic month from the cycle of 251 synodic months and the refined Babylonian value for the number of days in an anomalistic month:

$$(269/251) \cdot 27;33,16,30\text{d} = 29;31,50,11,36\text{d}.$$

He then averaged the two, and got 29;31,50,8,15d, adding that "such an origin, however, would make [the accuracy of this parameter] wholly fortuitous, which is difficult to accept in light of the consistent excellence of System B parameters generally".[33] Before accepting this reconstruction, it would be helpful to see if there are other examples of averaging parameters in ancient astronomy, and to offer a motive for averaging in this case (since either value would seem to be acceptable).

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6. J. P. Britton, "An early function for eclipse magnitudes in Babylonian astronomy", *Centaurus*, xxxii (1989), 1–52, esp. pp. 4–13. As Britton notes (pp. 2–3), the limits of the nodal zone for lunar eclipses (within which the lunar latitude is sufficiently small for a lunar eclipse to occur) vary from 9.5° to 12.2° before or after a node and, since the nodal elongation of the sun increases by roughly 30.7° per month, lunar eclipses cannot occur in consecutive months. There are two nodal zones, one about the ascending node, and the other about the descending node. After 6 months, the nodal elongation of the sun accumulates to about 184°, and so it moves from one nodal zone to the next, i.e., its nodal elongation from the second node is about 4° greater than it was from the initial node; hence, the sun will again be in a nodal zone unless it was too close to the end of the initial nodal zone. When the sun is in the nodal zone near one of the limits (at which time the moon is 180° from the sun), the eclipse may not be noticed because of its small magnitude. See also A. Aaboe, "Remarks on the theoretical treatment of eclipses in Antiquity", *Journal for the history of astronomy*, iii (1972), 105–18. For historical background in support of Britton's interpretation of the data, see Steele, *Eclipse times* (ref. 1), 78ff.
7. Aaboe *et al.*, *Saros cycle* (ref. 5), 14.
8. See K. P. Moesgaard, "The Full Moon Serpent: A foundation stone of ancient astronomy?", *Centaurus*, xxiv (1980), 51–96. *Cf.* Britton, "Column Φ" (ref. 4), 34, n. 4.
9. Aaboe *et al.*, *Saros cycle* (ref. 5), 18–20.
10. W. M. Smart, *Text-book on spherical astronomy* (Cambridge, 1965), 420.
11. *Ibid.*
12. Brack-Bernsen, "Babylonische Mondtexte" (ref. 4), 354.
13. For the contrary claim, see Britton, "Lunar anomaly" (ref. 4), 192f.
14. F. Rochberg-Halton, *Aspects of Babylonian celestial divination: The lunar eclipse tablets of Enūma Anu Enlil* (Vienna, 1988).
15. O. Neugebauer, *A history of ancient mathematical astronomy* (Berlin and New York, 1975), 484ff.
16. Britton, "Column Φ" (ref. 4), 25f; *idem*, "Lunar anomaly" (ref. 4), 211f. Britton believes that the relationship, 251 synodic months = 269 anomalistic months, was known to the author(s) of System A, although it does not occur explicitly there (privately communicated).
17. Brack-Bernsen, "Babylonische Mondtexte" (ref. 4), 349.
18. Sachs and Hunger, *Astronomical Diaries* (ref. 3), i, 20.
19. Brack-Bernsen, "Babylonische Mondtexte" (ref. 4), 354; *eadem*, "Goal-Year Tablets" (ref. 4), 154–7; *eadem*, *Entstehung* (ref. 4), 61–68, 133.
20. For a justification of these assumptions, see Brack-Bernsen, "Goal-Year Tablets" (ref. 4), 166ff; *cf. eadem*, *Entstehung* (ref. 4), 131f.
21. Brack-Bernsen, "Babylonische Mondtexte" (ref. 4), 352.
22. See Brack-Bernsen, "Babylonische Mondtexte" (ref. 4), 349.
23. B. R. Goldstein, "The obliquity of the ecliptic in ancient Greek astronomy", *Archives internationales d'histoire des sciences*, xxxiii (1983), 3–14, esp. p. 8; and A. C. Bowen and B. R. Goldstein, "Hipparchus' treatment of early Greek astronomy," *Proceedings of the American Philosophical Society*, cxxxv (1991), 233–54, esp. pp. 237f and 248f. Although the Babylonians did not appeal to deductive demonstrations, their facility with numbers was certainly on a par with the

Greeks, and this theorem was well within their grasp.

24. It is difficult to believe that these month lengths were derived independently and, while it is tempting to consider that the value for the synodic month was derived directly from the value for the anomalistic month, this seems unlikely, for (as noted in Appendix 2):

$$(269/251) \cdot 27;33,16,30 = 29;31,50,11,36\text{d}$$

rather than 29;31,50,8,20d.

25. Britton, "Column Φ" (ref. 4), 24. Britton ("Lunar anomaly" (ref. 4), 242f) remarks that this value for the anomalistic month is the "sole month-length in the System A theory that is explicitly defined".
26. O. Neugebauer, *"Saros" and lunar velocity in Babylonian astronomy* (*Danske Videnskabernes Selskab, Matematisk-fysiske Meddelelser*, xxxi/4; Copenhagen, 1957), 12ff; *cf.* Neugebauer, *History* (ref. 2), 501. Neugebauer considered a unit of 1H such that 6H = 1d, whereas the Babylonians used a unit, corresponding to time-degrees, called *ush* such that 360 *ush* = 1d, i.e., 1H = 60 *ush*. Judging from Britton's remark (see ref. 25), this value for the synodic month played no role in the theory of System A.
27. Britton has withdrawn the suggestion he made previously ("Column Φ" (ref. 4), 24f): privately communicated.
28. For the step in the Babylonian computation where the error occurs, see Neugebauer, "*Saros*" (ref. 26), 13, n. 16.
29. See, e.g., G. J. Toomer, *Ptolemy's Almagest* (New York and Berlin, 1984), 176, n. 10; and A. Aaboe, "On the Babylonian origin of some Hipparchian parameters", *Centaurus*, iv (1955), 122–5.
30. Britton, "Lunar anomaly" (ref. 4), 211ff.
31. If the upper bound is taken to be the value for the synodic month in the Saros text where m = 29;31,50,19,11,4,56d, then $223m \approx$ 6585;20,1,18,... d and the result is slightly higher: 29;31,50,7,20d.
32. Britton, "Column Φ" (ref. 4), 25.
33. Britton, "Lunar anomaly" (ref. 4), 249, n. 21.