



Summer Meeting of the Association for Symbolic Logic

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SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Summer meeting of the Association for Symbolic Logic was held on Friday, September 2, 1959 at the University of Utah in Salt Lake City, Utah in conjunction with the Summer meetings of the American Mathematical Society and the Mathematical Association of America.

Professor H. Curry of the University of Pennsylvania delivered the invited address, in the afternoon, on *The combinatory structure of grammar*.

At the morning session, presided over by Professor Frederic B. Fitch, three twenty minutes papers were delivered. Five papers were delivered by title.

HUGO RIBEIRO

HASKELL B. CURRY. *The combinatory structure of grammar.*

In the study of combinatory logic, the author has found it expedient to introduce a notion of functionality. This is introduced via a constant (called an "ob") \mathbf{F} , such that, if α and β are any categories $\mathbf{F}\alpha\beta$ is the category of functions from α into β , i.e. of functions f such that for any x in α , fx is in β . This notion can be interpreted in terms of linguistics (see Curry and Feys, *Combinatory logic*, Vol. 1 (1958), pp. 264–265, 274–275). This suggests that this notion would be useful in forming a theory of the grammars of the natural languages. For such a purpose the "ob" formal systems of the author are more suitable than the concatenative systems or calculuses used by most logicians. By such means what the linguists call "phrase-structure grammars" and "transformation grammars" can be treated from a unified point of view.

JOSEPH D. RUTLEDGE. *On the definition of an infinitely-many-valued predicate calculus.*

A formal definition is given of the infinitely-many-valued predicate calculus discussed by Rosser at the January 1959 meeting of the Association. It is shown by a topological argument that the set of theorems of this system is just the set-theoretic limit of the sequence of theorem-sets of the n -valued Łukasiewicz predicate calculi as n goes to infinity. From this it follows that this system can be defined equally well by means of denumerably and non-denumerably infinite value sets, with corresponding restrictions on the predicates accepted. Thus this system is shown to be the only natural extension of the Łukasiewicz predicate calculi to infinitely many values, and of the infinitely-many-valued Łukasiewicz propositional calculus to a first order predicate calculus.

STEVEN OREY. *Relative interpretations.*

Feferman XXII 106, 107 will be referred to as [F]. Our results use those of [F] extensively. The notation is that of [F]; so for simplicity only theories with constants $\mathbf{0}$, \mathbf{S} , $+$, \cdot , are considered. Let A , B be two such theories which are consistent and axiomatizable and such that all theorems of P are provable in A . Let \mathcal{J} be the set of relative interpretations of B into A .

Theorem 1. *Let φ be a sentence of B . Then the following three conditions are equivalent.*

(a) *For some $F \in \mathcal{J}$, $\vdash_A F\varphi$.* (b) *There exists a $\beta(x)$ defining A_{β}^* in A such that $\vdash_A \text{Pr}_{\beta}(\sim\varphi)^*$.* (c) *There exists an n and $F_1, \dots, F_n \in \mathcal{J}$ such that $\vdash_A F_1\varphi \vee \dots \vee F_n\varphi$.*

Theorem 2. *Let φ be a sentence of B . Then the following five conditions are equivalent.*

(a) *For some $F, G \in \mathcal{J}$ $\vdash_A F\varphi$, $\vdash_A \sim G\varphi$.* (b) *For every sentence η of A there is an $F \in \mathcal{J}$ such that $\vdash_A F\varphi \leftrightarrow \eta$.* (c) *For every $F \in \mathcal{J}$ there is a $G \in \mathcal{J}$ such that $\vdash_A F\varphi \leftrightarrow \sim G\varphi$.*

(d) For some $F \in \mathcal{F}$ there is a $G \in \mathcal{F}$ such that $\vdash_A F\varphi \leftrightarrow \sim G\varphi$. (e) There exists β defining A_B^* in A such that $\vdash_A \sim \text{Pr}_\beta(\overline{\varphi^*}) \wedge \sim \text{Pr}_\beta(\overline{(\sim\varphi)^*})$.

Theorem 3. If all theorems of B are theorems of A there exists a φ satisfying the conditions of Theorem 2.

Theorem 4. If for every finitely axiomatizable subtheory C of B $C \leq A$ then $B \leq A$.

Theorem 5. There exists a non-axiomatizable theory D such that A is a sub-theory of D , $D \not\leq A$, but $C \leq A$ for every axiomatizable subtheory C of D .

Theorem 6. For any β defining A_B^* in A one can find an α defining A_A^* in A such that $\vdash_A \text{Con}_\alpha \rightarrow \text{Con}_\beta$. If β is a $\mathcal{P}\mathcal{R}$ -definition and A_A^* is primitive recursive α can be chosen to be a $\mathcal{P}\mathcal{R}$ -definition.

In connection with Theorem 6 cf. Kreisel XXIII 108.

Thanks are due to Feferman for pointing out that an additional hypothesis in the theorems above, namely that A is ω -consistent, is unnecessary in view of a recent result by him and Ehrenfeucht.

STEVEN OREY. *Faithful relative interpretations.*

The notation is that of the preceding abstract. A and B are as above. Let $\beta(x)$ define A_B^* in A and suppose $\vdash_A \text{Con}_\beta$. Then by Theorems 5 of [F] \mathcal{F} is not void. Call $F \in \mathcal{F}$ *faithful* if for every sentence φ $\vdash_B \varphi$ if and only if $\vdash_A F\varphi$.

The proof of Theorem 5 of [F] proceeds by arithmetizing a proof of the completeness theorem. The crucial step is extending Ax_B to a maximal consistent set Ω by the Lindenbaum procedure. Formally one constructs a formula $\gamma(x)$ corresponding to Ω such that $\vdash_A \alpha(x) \rightarrow \gamma(x)$, $\vdash_A \text{Con}_\gamma$, and the formalization of "for every formula φ $\gamma(\overline{\varphi^*})$ or $\gamma(\overline{(\sim\varphi)^*})$ " is provable in A . Such a γ at once leads to an $F \in \mathcal{F}$ such that $\vdash_A F(\theta) \leftrightarrow \gamma(\overline{\theta^*})$ for every sentence θ of B . So to obtain a faithful $F \in \mathcal{F}$ one must construct γ so that $\vdash_A \gamma(\overline{\theta^*})$ only if $\vdash_B \theta$. To this end one can try to formalize a proof of the completeness theorem which instead of using Lindenbaum's procedure completes Ax_B thus:

Let φ_n be the n^{th} formula of B in some ordering. Let Ω_0 be Ax_B . Let Ω_{n+1} be $\Omega_n \cup \{\varphi_{n+1}\}$ ($\Omega_n \cup \{\sim\varphi_{n+1}\}$) if $\vdash_{\Omega_n} \varphi_{n+1}$ ($\vdash_{\Omega_n} \sim\varphi_{n+1}$); if this leads to no decision flip a coin and let Ω_{n+1} be $\Omega_n \cup \{\varphi_{n+1}\}$ ($\Omega_n \cup \{\sim\varphi_{n+1}\}$) if it lands heads (tails).

In formalizing the above argument the condition that at the n^{th} stage the coin shows heads must be replaced by " $\psi(\bar{n})$ holds", where $\psi(x)$ is a suitable formula of A . Writing θ^0 (θ^1) for θ ($\sim\theta$) the precise condition $\psi(x)$ must satisfy is that for all formulas φ of B , all positive integers n , and all n -tuples of 0's and 1's (i_0, \dots, i_{n-1}), $\vdash_A \psi(0)^{i_0} \wedge \dots \wedge \psi(n-1)^{i_{n-1}} \rightarrow \text{Pr}_\beta(\overline{\varphi^*})$ only if $\vdash_B \varphi$.

Example: If P_c is P with one new constant, c , the formula asserting that the n^{th} prime divides c can serve for $\psi(\bar{n})$. One can show: $B \leq P$ if and only if $B \leq P_c$ if and only if B is faithfully relatively interpretable in P_c .

NUEL D. BELNAP, JR. *Pure rigorous implication as a "Sequenzen-kalkül".*

As a partial solution to the problem of formulating a Gentzen *Sequenzen-kalkül* equivalent to the system E of entailment (Anderson and Belnap, *A modification of Ackermann's "rigorous implication"* [abstract], XXIII 457), we here present a *Sequenzen-kalkül*, LI, equivalent to the pure implicational fragment of E as defined by axioms (1)–(4) and rules (α) and (δ) of Ackermann XXII 327.

The postulates of LI are (i) a set of *prime statements* of the form $A \Vdash A$; (ii) a rule of *contraction*, from $\Gamma, A, A, \Delta \Vdash C$ to infer $\Gamma, A, \Delta \Vdash C$; (iii) a rule for *implication on the right*, from $\Gamma, A \Vdash B$ to infer $\Gamma \Vdash A \rightarrow B$; and (iv) a rule for *implication on the left*, from $\Gamma, \Delta \Vdash A$ and $\Phi, B, \Psi \Vdash C$ to infer $\Pi, \Delta, \Psi \Vdash C$, where Π contains as constituents $A \rightarrow B$ together with all and only the constituents of Γ and Φ , and the

constituents of Γ and Φ have the same order in Π that they have in Γ and Φ respectively; and where Δ is non-empty iff A is a propositional variable.

There is provable for LI an appropriate Elimination Theorem (*Hauptsatz*): if $\Gamma, \Delta \vdash A$ and $\Phi, \Psi \vdash C$ are theorems of LI, then so also is $\Pi, \Psi^* \vdash C$, where all and only the constituents of Γ and Φ appear in Π , and have the same order in Π which they have in Γ and Φ respectively; and where Ψ^* must contain A , Δ is non-empty iff A is a propositional variable, and Ψ^* is the result of replacing A by Δ throughout Ψ . The proof is by induction on rank and degree as in Gentzen.

With the help of the Elimination Theorem, it is easily shown that LI is equivalent to the pure implicational fragment of E as defined above.

Remarks: (1) The interpretation of $A_1, A_2, \dots, A_n \vdash B$ is *not* that the conjunction of A_1, A_2, \dots, A_n entails B ; this statement has rather the stronger sense that A_1 entails that A_2 entails that $\dots A_n$ entails B . (2) In accordance with (1), LI has no rule permitting addition of arbitrary constituents, nor any rule permitting arbitrary permutation of constituents — though some of the functions of the usual rule of permutation are carried in the statement of the rule for implication on the left. (3) As yet no decision procedure for LI has been found; the usual methods depend on the presence of the converse of the rule of contraction, which fails for LI. (4) The condition that $\vdash A \rightarrow B$ only if A and B have some propositional variable in common can be taken as a partial explication of the intuitive demand for *relevance* as between antecedent and consequent of an entailment. LI satisfies this condition. (5) The usual rules for conjunction and disjunction may be added to LI, an Elimination Theorem being provable for the system so defined; but for this system the distributivity principles for conjunction and disjunction (present in E) are not forthcoming.

P. C. GILMORE. *An alternative to set theory.*

Because the concept of set is so simple and yet so powerful, its convenience for mathematicians is overwhelming. The concept, however, must be limited by *ad hoc* restrictions if the set theoretic paradoxes are to be avoided. An alternative to set theory is proposed which is as simple in conception as naive set theory and yet which is not troubled by its inherent contradictions.

In place of a universe of sets and the membership relation over the universe there appears a universe of symbols and an epsilon and nu relation over the universe. Symbols such as '1', '2', '3', 'even', '<', '+', etc., can be understood to be members of the universe of symbols. The sentence '2 is even' is understood to assert that the symbol '2' is epsilon related to the symbol 'even', and the sentence '3 is not even' is understood to assert that the symbol '3' is nu related to 'even'. Thus sentences '3 is even' and '3 is not even' are not negations of one another although, accepting $(x)(y) \sim (x \epsilon y \ \& \ x \nu y)$ as true, each implies the negation of the other.

Relations and functions are conceptually much simpler in the new theory than in set theory. '<', for example, is a relation symbol because, although nothing is epsilon or nu related to it, there are symbols epsilon or nu related to the concatenation of it with other symbols. Thus '2' is epsilon related to '<3' and nu related to '<1'. Similarly '(1 + 1)' is a symbol which is epsilon related to '<3' and nu related to '<1'.

Although the problem of the consistency of axiomatic theories motivated by the new theory is no less difficult than with axiomatic set theories, nevertheless greater confidence can be given to the new theory than to set theory since the paradoxes which are inherent in the concept of set are avoided in the new theory. Thus despite that there can be no set of all sets which are not members of themselves, there can be a symbol R for which $(x) ((x \epsilon R \equiv x \nu x) \ \& \ (x \nu R \equiv x \epsilon x))$ is true, although it necessarily follows that $(\sim R \epsilon R \ \& \ \sim R \nu R)$ is true also.

G. KREISEL. *Inessential extensions of Heyting's arithmetic by means of functionals of finite type.*

Let T denote the system given by Gödel in *Dialectica* 47/48 (1958), pp. 283–284; T_1 is obtained by adding to the non-logical axioms of T an intuitionistic predicate calculus of type ω and the induction schema for all formulae in the extended notation. T_1^e is obtained from T_1 by adding the axioms of choice $(x)(\exists y)\mathcal{A}(x, y) \rightarrow (\exists Y)(x)\mathcal{A}[x, Y(x)]$ for all finite types of x and y and \mathcal{A} in the notation of T_1 . F' is Gödel's translation l.c. of F , extended to $T_1(T_1^e)$, F^* is the author's translation of F , given in para. 3.52 on p. 112 of *Constructivity in Mathematics* (1959), which we denote by CM.

Theorem 1 (mild extension of Gödel's result, l.c.): *If $\vdash_1^e (F \rightarrow G)$ then $F' \vdash G'$.*

Corollary 1. If F is a conjunction of formulae $(x)\neg(y)\neg A(x, y) \rightarrow (x)(Ey)A(x, y)$, A quantifier-free, x and y of arbitrary type, and $\vdash_1^e (F \rightarrow G)$ then $\vdash G'$, since F' is an identity.

Theorem 2. *If $\vdash_1^e (F \rightarrow G)$ then $\vdash_1 (F^* \rightarrow G^*)$.* *Corollary 2.* If F is a conjunction of substitution instances of (a) the formula $[\neg p \rightarrow (q \vee r)] \rightarrow [(\neg p \rightarrow q) \vee (\neg p \rightarrow r)]$, or (b) $(x)\mathfrak{A}(x) \rightarrow (\exists y)\mathfrak{B}(x, y) \rightarrow (x)(\exists y)[\mathfrak{A}(x) \rightarrow \mathfrak{B}(x, y)]$ for negative \mathfrak{A} and \mathfrak{B} and x and y of arbitrary type, and if $\vdash_1^e (F \rightarrow G)$ then $\vdash_1 G^*$, since F^* is an identity.

Corollary 2(b) implies (for quantifier-free A) that $\vdash_1^e [\neg(y)\neg A(y) \rightarrow (Ey)A(y)]$ if and only if $\vdash_1 (y)\neg A(y)$ or $\vdash_1 (Ey)A(y)$, which, by Gödel's incompleteness theorem, is not true for all such A . This shows, first, that *the converse to Theorem 1 is false*, and, second, that *the conjecture of CM, p. 105, para. 2.11 is false* since (in the notation of CM) $\neg(\varphi)A(\varphi) \rightarrow (E\varphi)\neg A(\varphi)$, with recursive A , is derivable in T_1^e from substitution instances of 2(b) with universal formulae \mathfrak{A} and \mathfrak{B} only if $\vdash_1 (\varphi)A(\varphi)$ or $\vdash_1 (E\varphi)\neg A(\varphi)$. By Theorem 1, the formula of 2(a) with $(x)[\alpha(x) \neq 0 \ \& \ \beta(x) \neq 0]$ for p , $(Ex)[\alpha(x) = 0]$ for q , $(Ex)[\beta(x) = 0]$ for r , is not provable in T_1^e and so *the converse to Theorem 2 is false*.

Theorem 3. *If F is a formula of Heyting's arithmetic then $\vdash_1 F$ just in case $\vdash_{HA} F$.*

The proof constructs a 'model' in HA of every finite subsystem of T_1 by relativizing the quantifiers of higher type to effective operations in the sense of CM, p. 117, para. 4.2. *Corollary 3.* If F is $(x)\neg(y)\neg A(x, y)$ with quantifier-free A , $\vdash_1^e F$ just in case $\vdash_{HA} (x)(Ey)A(x, y)$; for if $\vdash_1^e F$, by Theorem 1, $\vdash A[x, Y(x)]$, hence, without axioms of choice, $\vdash_1 (x)(Ey)A(x, y)$, and, by Theorem 3, $\vdash_{HA} (x)(Ey)A(x, y)$. This generalizes this JOURNAL, vol. 23 (1958), p. 172, Remark 6.1.

The proofs of Theorems 1–3 and Corollaries 1–3 are finitist.

G. KREISEL. *Inessential extensions of intuitionistic analysis by functionals of finite type.*

We use the notation of the preceding abstract. T_2 is obtained from T_1 by adding (i) the axiom of choice for lowest type, and (ii) the fan theorem, i.e., (3) and (4) on p. 286 of CM in Kleene's formalization KA of intuitionistic analysis, and $T^B(T_1^B)$ is obtained from $T(T_1)$ by adding a constant $Z_0(x, y)$ with an axiom B which expresses that Z_0 is a modulus of continuity of functionals x applied to functions bounded by y , e.g., $(s)[s \leq Z_0(x, y) \rightarrow y'(s) = y''(s) \leq y(s)] \rightarrow x(y') = x(y'')$.

Theorem 1 (cf. Gödel, l.c., p. 286, last lines): *If $\vdash_2^{eB} F$ then $\vdash^B F'$.*

Theorem 2. *If $\vdash_1^{eB} F$ then $\vdash_1^B F^*$.*

Theorem 3 (inessential extension): *If F is a formula of KA, then $\vdash_2^B F$ if and only if $\vdash_{KA} F$.*

The proof of Theorem 3 constructs a 'model' in KA for each finite subsystem of T_2^B by relativizing the functionals of higher type to neighbourhood functions, introduced in CM, pp. 114–116. The corollaries 1–3 of the preceding abstract, which are there stated for extensions of HA, apply to extensions of KA: replace \vdash_1^e by \vdash_2^{eB} , \vdash_1 by \vdash_1^B , and \vdash by \vdash^B .

Theorems 1 and 2 supply finitist *consistency proofs* for intuitionistic analysis with the fan theorem and axioms of choice, relative to the systems T^B and T_1^B . The latter systems are valid when the logical constants are interpreted classically and the functionals range over continuous functionals in the sense of CM. Classically, consistency proofs for extensions of T_2 are needed because the fan theorem is false under the classical interpretation of the logical constants. Also T_1^{cB} requires a consistency proof on the classical interpretation, because the axioms of choice applied to discontinuous predicates $\mathcal{A}(x, y)$ are not satisfied by continuous functionals, while B is not satisfied when x ranges over discontinuous functionals. In contrast, T_1^c is a valid system under the classical interpretation of the logical constants and the class of arbitrary functionals as the range of the variables of higher type.

The corollaries 1 and 2 are primarily of intuitionistic interest. Though generally the formulae F there considered are not intuitionistically valid, the consequences G obtained from F in T_2^{cB} are, provided only G is such that, in Corollary 1, $G' \rightarrow G$ holds (intuitionistically), e.g., if G is prenex, or, in Corollary 2, $G^* \rightarrow G$ holds, e.g., if G is negative. In another direction, we have the following application of Theorem 2: Call Heyting's predicate calculus (HPC) *strongly complete* (cf. XXIII, p. 319) for the sentence $\mathfrak{A}(P_1, \dots, P_k)$, with respect to denumerable domains, if $(P)(P_1) \dots (P_k)\mathfrak{A}_P(P_1, \dots, P_k) \rightarrow (\exists y)$ [y is the number of a proof of ' $\mathfrak{A}(P_1, \dots, P_k)$ ' in HPC]; the individual variables in $\mathfrak{A}_P(P_1, \dots, P_k)$ range over natural numbers which satisfy P. For each \mathfrak{A} of HPC, strong completeness w.r.t. denumerable domains can be stated in the notation of T_2 , and can be proved in T_2^{cB} for negative \mathfrak{A} just in case $\vdash_{\text{HPC}} \mathfrak{A}$ is decidable in T_2^{cB} . This sharpens a result given in a recent abstract by the author (XXIII, pp. 456-457).

ALAN ROSS ANDERSON and NUEL D. BELNAP, JR. *A proof of the Löwenheim-Skolem theorem.*

Γ, Δ , and Λ are sequences of wffs: Γ and Δ are finite or empty, Λ is infinite or empty. If S is the sequence $\Gamma, A, \Delta, \Lambda$ then (i) A is wff of S, and (ii) if A has the form $\bar{p}, \bar{p}, f(x_1, \dots, x_n)$, or $\overline{f(x_1, \dots, x_n)}$, then A is an atom of S. If S contains atoms A and \bar{A} , then S is an explicit contradiction.

Let S be a sequence of wffs, containing as connectives only \neg, \wedge , and $\forall x$, such that no variable occurs both bound and free in S, and such that infinitely many individual variables fail to occur in S. $V (= \{v_1, \dots, v_i, \dots\})$ is the sequence of variables (in alphabetical order) which do not occur bound in S. The *satisfiability tree* for S consists of all those *satisfiability branches* \mathbf{B} such that S is the first member S_1 of \mathbf{B} , and if S_i is the i -th member of \mathbf{B} , then: (i) if S_i contains only atoms, or is an explicit contradiction, then S_i is the terminus of \mathbf{B} ; (ii) otherwise S_i has the form $\Gamma, A, \Delta, \Lambda$, where A is the leftmost non-atomic wff of S_i , and if A has the form (1) \bar{B} , (2) $B \wedge C$, (3) $\overline{B \wedge C}$, (4) $\forall yBy$, (5) $\overline{\forall yBy}$, then S_{i+1} is respectively (1) $\Gamma, B, \Delta, \Lambda$, (2) $\Gamma, B, C, \Delta, \Lambda$, (3) $\Gamma, \bar{B} \{ \text{or } \bar{C} \}, \Delta, \Lambda$, (4) $\Gamma, Bv_j, \Delta, \forall yBy, \Lambda$, where v_j is the first variable in V such that Bv_j does not occur as a wff of S_1, \dots, S_i , and where Λ is empty if S_i is finite, and Δ contains i members if S_i is infinite, (5) $\Gamma, \overline{Bv_j}, \Delta, \Lambda$, where v_j is the first variable in V not occurring free in S_1, \dots, S_i .

The satisfiability tree for S has the following properties. (By an atom or wff of a branch \mathbf{B} , we mean an atom or wff of some S_i in \mathbf{B} .) (I) If every branch terminates in an explicit contradiction, then the wffs of S are not simultaneously satisfiable (the tree constituting a "proof" of this). (II) If some branch \mathbf{B} does not terminate in an explicit contradiction, then (a) if \bar{A} occurs as an atom of \mathbf{B} , then A does not; (b) if (1) \bar{A} , (2) $A \wedge B$, (3) $\overline{A \wedge B}$, (4) $\forall yBy$, (5) $\overline{\forall yBy}$, occur as wffs of \mathbf{B} , then so do (1) A , (2) A and B, (3) either \bar{A} or \bar{B} , (4) Bv_i for every v_i in V , (5) $\overline{Bv_i}$ for some v_i in V .

Now let C be a class of simultaneously satisfiable wffs. Rewrite them equivalently in \neg , \wedge , and $\forall x$, in such a way that no variable occurs both bound and free in C , and infinitely many individual variables fail to occur in C , and arrange the resulting wffs in a sequence S . Then by (I), there is some branch \mathbf{B} of the satisfiability tree for S which does not terminate in an explicit contradiction. To the free variables of this branch \mathbf{B} assign values as follows: to p give t iff p is an atom of \mathbf{B} ; to v_i give i ; to n -ary f give the function taking (i_1, \dots, i_n) into t iff $f(v_{i_1}, \dots, v_{i_n})$ is an atom of \mathbf{B} . Then by (II) and an induction on the length of A , every wff A of \mathbf{B} (and in particular every wff of S) takes the value t by this assignment.

Hence the wffs of S (and those of C) are simultaneously satisfiable in the positive integers.