A. N. Prior. *Notes on a group of new modal systems.* ***Logique et analyse,*** n.s. vol. 2 (1959), pp. 122–127.

R. A. Bull. *An axiomatization of Prior's modal calculus* **Q**. ***Notre Dame journal of formal logic,*** vol. 5 no. 3 (1964), pp. 211–214.

A. N. Prior. *Axiomatisations of the modal calculus* **Q**. Ibid., pp. 215–217.

In XXV 342 Prior gave a matrix for a system (**Q**) of modal logic in which the values of the formulas are infinite sequences containing 1's, 2's, and 3's or combinations of these and beginning with 1 or 3. The designated sequences are those not containing 3's. All functors take 2 into 2, the truth functors treat 1 and 3 as truth values, Lp has 1 throughout iff p has 1 throughout, otherwise 3 except where p has 2 and if p has 1 anywhere then Mp has 1 everywhere except where p has 2, otherwise 3 except where p has 2. The interpretation of **Q** envisages propositions as being, at different times (or in different "possible worlds"), either true, false, or unstable. Bull proves that **Q** with **L** and M as primitive (and $L =_{df} NMN$) can be axiomatized by adding to PC the axioms and rules: 1. $CLpp$, 2. $C\mathbf{L}pp$, 3. $CK\mathbf{L}p\mathbf{L}q\mathbf{L}Kpq$, **RQ**$La$. $C\beta\gamma \Rightarrow C\beta L\gamma$ (where (1) β is fully modalized and (2) the variables of β each occur in γ), **RQ**Lb. $C\mathbf{L}\alpha C\beta\gamma \Rightarrow C\mathbf{L}\alpha C\beta L\gamma$ (where (1) β is fully modalized and (2) the variables of β each occur in α or γ), RQ**L**. $C\mathbf{L}\alpha C\beta\gamma \Rightarrow C\mathbf{L}\alpha C\beta\mathbf{L}\gamma$ (where (1) β is fully modalized and (2) the variables of β and γ each occur in α).

The proof is by reduction to a normal form which gives the corollary that **Q** is decidable. In Prior's first paper he introduces an operator S to mean "is necessarily statable." I.e., Sp is to have 1 everywhere if p has no 2's and otherwise 3 except where p has 2's. Prior gives some axioms for S which seem to him to reflect the properties of this matrix. These are: $A1$. $CLpp$, **RS**1. $\vdash CS\alpha Sp$ (where p is any variable in α), **RS**2. $CSpCSq \cdots S\alpha$ (where p, q, etc. are all the variables in α), and **RSL**. $\vdash C\alpha\beta \rightarrow \vdash CSpCSq \cdots C\alpha L\beta$ (where α is fully modalized and p, q, etc. are all the variables in β that are not in α).

In the second paper Prior shows that with $\mathbf{L}p$ defined as $KSpLp$ these postulates are sufficient to derive Bull's, together with the two implications $CSp\mathbf{L}Cpp$ and $C\mathbf{L}CppSp$. This shews that Prior's axiomatization is strong enough for **Q**, though it does not shew, as Prior appears to suppose, that it is not too strong. One needs to derive Prior's basis in Bull's system.

In his first paper Prior had thought that his axiomatization for **Q** was equivalent to a basis conjectured by Lemmon, but he admits in the second paper that this was not in fact proved. There is also in the first paper some discussion of extensions of **Q**. M. J. Cresswell

R. B. Angell. *A propositional logic with subjunctive conditionals.* ***The journal of symbolic logic,*** vol. 27 no. 3 (for 1962, pub. 1963), pp. 327–343.

In attacking the problem of providing a formal propositional logic for the subjunctive conditional "if it were the case that p then it would be the case that q," the author offers a system A1 which is *not* a fragment of the two-valued calculus. In particular, the non-tautologous formula $(p \rightarrow q) \rightarrow -(p \rightarrow -q)$, called "the principle of subjunctive contrariety," is defended as a plausible truth about subjunctive conditionals.

Grammar of A1: "$-$", "$.$", and "$\rightarrow$" (for negation, conjunction, and the subjunctive conditional).

Axioms: $((q \rightarrow r) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)))$, $((p \rightarrow q) \rightarrow ((r.p) \rightarrow (q.r)))$, $((p \rightarrow -(q.r)) \rightarrow ((q.p) \rightarrow -r))$, $((p.(q.r)) \rightarrow (q.(p.r)))$, $((p \rightarrow -q) \rightarrow (q \rightarrow -p))$, $(--p \rightarrow p)$, $((p \rightarrow q) \rightarrow -(p.-q))$, $-((p.q).-p)$, $-(p.-(p.p))$, and $((p \rightarrow q) \rightarrow -(p \rightarrow -q))$.

Rules: modus ponens, adjunction, and substitution.

The system is shown consistent by means of a four-valued matrix. It is also shown to contain all tautologies in negation and conjunction, a large number of the usual tautologies involving the arrow, and some interesting non-tautologies (in addition to the principle of subjunctive contrariety) such as $-(q \rightarrow -q)$, $-((p \rightarrow q) \rightarrow (p \rightarrow -q))$, and $-((p \rightarrow q).(p \rightarrow -q))$.

The last formula shows that A1 is paradox-free in the sense that there is no strongest formula, and its contrapositive version $-((p \rightarrow q).(-p \rightarrow q))$ rules out a weakest. Standard paradoxical formulas such as $-p \rightarrow (p \rightarrow q)$ are ruled out by the four-valued matrix.

The matrix also excludes some old friends not ordinarily thought of as paradoxical, such as $(p.q) \rightarrow p$, $-p \rightarrow -(p.q)$, and even $p \rightarrow (p.p)$. The author discusses difficulties associated with the omission of these and some other standard tautologies, and concludes that although the system A1 itself is not altogether satisfactory, still its existence demonstrates the possibility of a logic including the principle of subjunctive contrariety. NUEL D. BELNAP, JR.

MIROSLAV MLEZIVA. *K axiomatisaci trojhodnotové výrokové logiky (On the axiomatization of three-valued propositional logic).* Czech with Czech, Russian, and English summaries. ***Časopis pro pěstování matematiky,*** vol. 86 (1961), pp. 392–403.

The author presents a system of three-valued propositional calculus with the (functionally complete) set of connectives $\{\rightarrow, R\}$. The system is characterized by the matrix (in which 1 is the "designated" element):

$\rightarrow$	0	$\frac{1}{2}$	1	R
0	1	1	1	0
$\frac{1}{2}$	1	1	1	1
1	$\frac{1}{2}$	0	1	$\frac{1}{2}$

The axioms given are: (1) $((p \rightarrow q) \rightarrow r) \rightarrow ((r \rightarrow p) \rightarrow (s \rightarrow p))$, (2) $R(p \rightarrow q) \rightarrow (Rq \rightarrow r)$, (3) $p \rightarrow ((q \rightarrow Rq) \rightarrow ((Rq \rightarrow q) \rightarrow R(p \rightarrow q)))$, (4) $p \rightarrow RRp$, and (5) $RRp \rightarrow p$; they are shown to be consistent, complete, and independent.

The system is of interest because Słupecki previously gave two systems S_3' and S_3'' of three-valued propositional calculus, each having a functionally complete set of three connectives (XIII 165). The connectives C, N, and R of Słupecki's S_3'' may be defined in terms of $\rightarrow$ and R as follows: $Cpq =_{\mathrm{df}} p \rightarrow (p \rightarrow q)$; $Np =_{\mathrm{df}} p \rightarrow Rp$; $Rp =_{\mathrm{df}} Rp$. ANN S. FEREBEE

EUGEN MIHĂILESCU. *Formele normale ale functorilor logicei clasice (Formules normales des foncteurs de la logique classique).* Roumanian with Russian and French summaries. ***Studii şi cercetări matematice,*** vol. 10 (1959), pp. 117–144.

Let $\mathscr{L}$ be the system defined by the following list of axioms (in which several misprints of the original paper have been corrected): *EEpqEqp, EEEpqrEpEqr, EAApqrApAqr, EAApqrAAqpr, EAEpqrEAprAqr, EAAppqApq, EAppp, EARpqrEEAprAqrr, EAKpqrAEEpqApqr, EACpqr AEApqqr, EADpqrARApqqr, EAHpqrARApqpr, EABpqrAEApqpr, EAGpqrAEEppqr, EAMpqr AERpqqr, EANprAERpqqr,* $EA\bar{A}pqrANApqr$, $EA\bar{K}pqrANKpqr$, $EA\bar{G}pqrANGpqr$, $EA\bar{M}pqrANM$ *pqr*. The rules of inference are the rule of substitution, and the rule from $E\alpha\beta$ and α to infer β.

Various theses are deduced and it is proved that $\mathscr{L}$ is incomplete since it has, besides theses and refutable formulas, also (infinitely many) free formulas. Here a formula is called *refutable* if its addition to the axioms would have the effect that all formulas are then deducible, and *free* if it is neither a thesis nor refutable. The free formulas are equipollent, in the sense that $E\alpha\beta$ is always a thesis if α and β are both free. And the shortest free formulas are *ENpp, NEpp, NCpp, Rpp, KNpp, Dpp, Hpp,* $\bar{A}Npp$, and *NBpp*.

Finally, the author shows that every formula can be reduced (as a consequence of the axioms) to a certain normal form, from which it can be seen whether or not it is a thesis of classical logic. S. RUDEANU

KIYOSHI ISÉKI. *An algebra related with a propositional calculus.* ***Proceedings of the Japan Academy,*** vol. 42 (1966), pp. 26–29.

YOSHINARI ARAI, KIYOSHI ISÉKI, and SHÔTARÔ TANAKA. *Characterizations of BCI, BCK-algebras.* Ibid., pp. 105–107.

KIYOSHI ISÉKI. *Algebraic formulation of propositional calculi with general detachment rule.* Ibid., vol. 43 (1967), pp. 31–34.

These articles present algebraic formulations for several implicational fragments of the propositional calculus. Earlier articles on *C-N* logics and related algebras were reviewed by Horn (XXXIII 625).

Several interesting implicational systems are distinguished by various combinations of (1) *CCpqCCqrCpr*, (2) *CpCCpqq*, (3) *Cpp*, (4) *CpCqp*, (5) *CCpCpqCpq*, and (6) *CCCpqpp* as axioms, with substitution and *modus ponens*. Thus (1), (4), and (6), which Iséki calls I, yield the implicational fragment of the full propositional calculus (Tarski-Bernays, 1928), containing all