As a contribution to the problem of change and immutability, the author introduces the notion of a "complete" (or dated) property such as green-in-August-1955. Such "properties" are of course immutable, if only for the reason that it makes no sense to talk about either acquiring or losing them.

Leaving it to the metaphysicians whether they find these contributions helpful or not, let me only point out that to the question "What was Napoleon doing on the 24th of August 1760?" all kinds of answers can be envisaged, but none of them the one which the author is ready to discuss, viz: "Napoleon does not exist on the 24th of August 1760." This string is ungrammatical in English. If it is meant to be a string in some constructed language with no tenses (but otherwise apparently similar to English), then the question is ungrammatical. The author, like so many others in similar situations, switches languages without mentioning or even noticing it.

One natural answer to the mentioned question would of course be: "Nothing. Don't you know that Napoleon was not even born in 1760?" If the questioner replies that he knows this all right but insists oı his question, he will be taken as a joker, idiot, or philosopher up to some trick.

The question of how to formalize this type of discourse is, of course, interesting and quickly leads into ground familiar to modern logicians. Y. Bar-Hillel

M. J. Cresswell. *On the logic of incomplete answers.* ***The journal of symbolic logic***, vol. 30 (1965), pp. 65–68.

The author defines the relation **Si**, where $\mathbf{Si}p(Qa)A(a)$ stands for "p is an incomplete answer to the question which a's satisfy A?" In the definition reference is made to (one-place) "functions of a propositional argument," which include not only the one-place truth-functions of two-valued logic but also the modal operators, belief operators, etc. The definition is:

$$\mathbf{Si}p(Qa)A(a) =_{\text{df}} : p \,.\, (f)[\{(a)fA(a) \,.\, (q)(r)((fq \,.\, fr) \supset f(q \,.\, r))\} \supset fp]$$

with quantification over propositions and functions of a propositional argument ("(f)").

Thus incomplete answers to $(Qa)A(a)$ would be conjunctions built up from true statements of the form $A(a_n)$. One of these incomplete answers can be at the same time the complete answer if the complete answer can be formulated as a finite conjunction.

Two theorems are proved: (1) $A(a) \supset \mathbf{Si}A(a)(Qa)A(a)$ (if a satisfies A then $A(a)$ is an incomplete answer to the question $(Qa)A(a)$); (2) $[\mathbf{Si}p(Qa)A(a) \,.\, \mathbf{Si}q(Qa)A(a)] \supset \mathbf{Si}(p \,.\, q)(Qa)A(a)$.

Although this formulation avoids a counter-intuitive aspect of Harrah's definition of *partial answer* (XXIX 136(2)), recourse to a non-extensional logic with all the functions of a propositional argument (a logic which has still to be developed formally) seems to the reviewer a high price to pay, especially when simple treatments in metalanguage are always possible. Gerold Stahl

Bolesław Sobociński. *A contribution to the axiomatization of Lewis' system S5.* ***Notre Dame journal of formal logic***, vol. 3 (1962), pp. 51–63.

Bolesław Sobociński. *On the generalized Brouwerian axioms.* Ibid., pp. 123–128.

Bolesław Sobociński. *A note on modal systems.* Ibid., vol. 4 (1963), pp. 155–157.

Ivo Thomas. *Solutions of five modal problems of Sobociński.* Ibid., vol. 3 (1962), pp. 199–200.

Ivo Thomas. *S1° and Brouwerian axioms.* Ibid., vol. 4 (1963), pp. 151–152.

Ivo Thomas. *S1° and generalized S5-axioms.* Ibid., pp. 153–154.

Ivo Thomas. *A final note on S1° and the Brouwerian axioms.* Ibid., pp. 231–232.

IVO THOMAS. *Modal systems in the neighbourhood of T.* Ibid., vol. 5 no. 1 (1964), pp. 59–61.

IVO THOMAS. *Ten modal models.* ***The journal of symbolic logic***, vol. 29 no. 3 (1964), pp. 125–128.

These articles consider a number of Lewis-like modal systems toward the weak end of the spectrum (thus adopting the advice of Lewis, XVI 225, p. 502), mostly inquiring into what happens when one or another of the formulas relating iterated modalities is added as a new axiom to one of these weaker systems.

It will be convenient to cast this review largely in terms of L (necessity), rather than M (possibility), though the articles differ in their primitive bases. C is material and $\mathfrak{C}$ strict implication, and K, A, and N are as usual. The various systems are definable in terms of the following rules and axioms: (1) *Fundamental postulates.* $\vdash CK\mathfrak{C}pqLpLq$; $\vdash L\varphi$ for every tautology φ; *modus ponens* for C; substitution; replacement of strict equivalents; from $\vdash L\varphi$ to infer $\vdash \varphi$. (2) *Additional axioms and rules.* (Trans) $= \mathfrak{C}K\mathfrak{C}pq\mathfrak{C}qr\mathfrak{C}pr$; ($\mathfrak{C}L$-dist) $= \mathfrak{C}\mathfrak{C}pq\mathfrak{C}LpLq$; $(LA) = \mathfrak{C}LpLApq$; (Gödel) = from $\vdash \varphi$ to infer $\vdash L\varphi$. (3) *Iterated modalities.* With L^n a sequence of n occurrences of "L", $(P_n) = CL^npL^{n+1}p$ $(n \geqq 1)$; $(B_n) = \mathfrak{C}pL^nMp$ $(n \geqq 1)$; $(A_{jk}) = \mathfrak{C}M^jpL^kMp$ $(j, k \geqq 1)$; $(D_{ijk}) = L^iCL^jLpL^kp$ $(0 \leqq i, j, k \leqq 1)$; $(R_{ijk}) = L^iCL^jCMpLqL^kCpq$ $(0 \leqq i, j, k \leqq 1)$.

The two weakest systems treated are Feys's $S1^o$ = Fundamental postulates + (Trans) and $S3^*$ = Fundamental postulates + ($\mathfrak{C}L$-dist) + D_{000}. Then $S2^o = S1^o + (LA)$, $S3^o = S1^o$ + ($\mathfrak{C}L$-dist), $S4^o = S1^o + (P_1)$, and $T^o = S1^o$ + (Gödel). All of these systems are alike in lacking $(D_{100}) = \mathfrak{C}Lpp$, and one obtains the usual systems S1–S4 (XVI 225) and T (III 120; = M of XVIII 174) by adding (D_{100}). Other systems are $S1^* = S1^o + (D_{111})$, $S1^+ = S1^o + (B_1)$, and $T^+ = T + (B_1)$. Then given any system Q, $Q_{P_n} = Q + (P_n)$ and $Q_{B_n} = Q + (B_n)$.

The whole series of articles is based on Sobociński$_1$ (i.e., the first in the above list of Sobociński articles), where it is shown that the usual axiomatizations of S5 with (A_{11}) or (B_1) are redundant with respect to (D_{100}). This is established by considering $S1^o$ and $S3^*$, which lack (D_{100}), and proving that either adding either (A_{11}) or (B_1) to $S3^*$ or adding (A_{11}) to $S1^o$ yields S5. (Adding B_1 to $S1^o$ yields $S1^+$, which contains $S2^o$.)

Sobociński$_2$ shows that in the context of any of $S1^*$ (which lies properly between $S1^o$ and S1), $S3^o$, or $S3^*$, the addition of any one of the formulas (B_n) yields S5, while Sobociński$_3$ considers the effect of adding one or another of the formulas (R_{ijk}) or (D_{ijk}) to one or the other of $S1^o$ or $S2^o$; for example, among the author's seventeen observations are that $S1^o + (R_{010})$ implies (D_{010}), and that $S2^o + (D_{011}) = S2^o + (R_{011})$. The connection between D_{ijk} and R_{ijk} can be seen somewhat more readily by writing the latter as $L^iCL^jALpLqL^kApq$.

In Thomas$_1$ various special independence questions raised in Sobociński$_1$ are uniformly answered in the affirmative by means of a four-element matrix. For example, it is shown that $S3^*$ does not contain $S1^o$.

Thomas$_2$ and Thomas$_4$ investigate the addition of generalized Brouwerian axioms (B_n) to $S1^o$; it turns out that certain pairs of axioms $\{(B_j), (B_k)\}$ yield S5 when added to $S1^o$, but that in the case of single axioms (B_n), even $T^o_{B_n}$ is weaker than even T. Independence results in Thomas$_4$ use Meredith's property calculus interpretation of modalities described below.

In a similar vein, Thomas$_3$ considers adding some generalized S5 axiom (A_{jk}) to $S1^o$: if $j = k$ or if $j = k + 2$, then the addition yields S5, while if $j = k + 2n$ for $n > 1$, or if $k = j + 2n$ for $n \geqq 1$, the result is equivalent to adding to $S1^o$ (B_{2n-2}) or (B_{2n}) respectively.

Rounding out the enterprise, Thomas$_5$ inquires about the addition of (P_n) to T, T^+, and $S1^+$: the systems T_{P_n} form a (descending) sequence between S4 ($= T_{P_1}$) and T, the systems $T^{+}_{P_n}$ a sequence between S5 (= $T^{+}_{P_1}$) and T^{+}, and the systems $S1^{+}_{P_n}$ a sequence between S5 (= $S1^{+}_{P_1}$) and $S1^{+}$. Independence results are by interpretations of modalities in terms of Prior-like (XXV 342) sequences.

Thomas$_6$ adds some new results of a like kind, but makes its chief contribution through its unifying use of the methods of C. A. Meredith's ***Interpretations of different modal logics in the 'property calculus'*** (August, 1958, recorded and expanded by A. N. Prior, mimeographed; Department of Philosophy, University of Canterbury). The central idea is that propositional expressions are construed as predicates, with $(N\varphi)x = N(\varphi x)$, $(C\varphi\beta)x = C(\varphi x)(\beta x)$, and $(L\varphi)x = \Pi y C U x y(\varphi y)$ for a constant relation-symbol U. The formal analogy of Meredith's interpretations via the constant U to those of Kripke (XXXI 276) via a relation of relative possibility between worlds or to those of Hintikka (XXIX 132) via a relation of alternativeness between model sets is extremely strong. But while the relational interpretations of Kripke and Hintikka are more intuitive and thereby more fit to play a role in completeness proofs, Meredith's is more direct and unencumbered with antecedent commitments as to what the field of the relation is to be. Common to all these approaches is the tying of modal reduction properties such as those expressed by (P_n) and (B_n) to simple properties of the relation. Thus, where U^n is the nth relative power of U, Thomas$_6$ considers (I) $\Sigma x U x y$, (II) Uxx, (III$_n$) $U^n \subseteq \breve{U}$, and (IV$_n$) $U^{n+1} \subseteq U^n$. Then any interpretation of U which satisfies (I) provides a model of T^{o}, while adding to (I) one of (IV$_n$), (III$_1$), (III$_1$, IV$_n$), or (III$_n$) provides a model for, respectively, $T^{o}_{P_n}$, $S1^{+}$ (= $T^{o}_{B_1}$), $S1^{+}_{P_n}$, and $T^{o}_{B_n}$. And (II) provides a model for T, while the same additions to (II) yield models for T_{P_n}, T^{+} (= T_{B_1}), $T^{+}_{P_n}$, and T_{B_n}. Thomas shows that all of these systems are distinct by providing a set of relational interpretations of U, thus summarizing in a neat way most of the containment and independence results of the preceding articles.

NUEL D. BELNAP, JR.

FEDERICO M. SIOSON. *Further axiomatizations of the Łukasiewicz three-valued calculus.* ***Notre Dame journal of formal logic***, vol. 5 no. 1 (1964), pp. 62–70.

In XVI 277(1), Alan Rose introduced some axiom systems for three-valued logic based on Łukasiewicz's negation N and an alternation operator A which was such that Łukasiewicz's Cxy could be defined as $ANxy$. The latter definition fails if A is interpreted as Łukasiewicz's alternation. In the present paper, the author considers some axiom systems for three-valued logic based on Łukasiewicz's negation N and a conjunction operator K which is such that Rose's Axy can be defined as $NKNxNy$ and Łukasiewicz's Cxy can be defined as $NKxNy$.

Under the given definition of Łukasiewicz's C, the author derives theorems in his axiomatic systems which correspond to the Wajsberg axioms for the three-valued logic of Łukasiewicz as given in 437*1*. Since Wajsberg's and the author's rules of inference are the same under the given definition of C, it is concluded that the Wajsberg system can be derived from the author's axiomatic systems.

Since it is known that Wajsberg's axiomatic system is adequate for Łukasiewicz's three-valued calculus, it is then claimed that the author's axiomatic systems are "adequate axiomatizations of the three-valued propositional calculus of Jan Łukasiewicz." It should be noticed, however, that Łukasiewicz's three-valued calculus and Wajsberg's axiomatization of it are both based on the choice of C and N as basic operators. On the other hand, the present author's axiom systems are based on the choice of K and N as basic operators. Hence, Łukasiewicz's C does not even occur in the author's systems. At best, the author's claim that his systems give an adequate axiomatization of Łukasiewicz's three-valued calculus can be justified only under the assumption of the author's definition of C and such a definition would be foreign and