itionistically in terms of the possibility of *constructing* an element which satisfies the predicate in question. This leads the author to an abstract theory of constructions with its own vocabulary, formulas, axioms, and rules of inference. The model-theoretic interpretation of a sentence in terms of (formal) set theory, in classical logic, is here parallelled by the association of a sentence of the predicate calculus with a corresponding formula in the calculus of constructions. The main theorem of the paper states that for every (formally derivable) theorem of Heyting's predicate calculus there is a corresponding provable formula in the theory of constructions.

Author's correction: Page 204, line 9, the λ-conversion is intended in the weak form for constants a only; otherwise it would conflict with the intended meaning, page 203, lines 7–8.

ABRAHAM ROBINSON

HAO WANG. *Process and existence in mathematics.* ***Essays on the foundations of mathematics, dedicated to A. A. Fraenkel on his seventieth anniversary,*** edited by Y. Bar-Hillel, E. I. J. Poznanski, M. O. Rabin, and A. Robinson for The Hebrew University of Jerusalem, Magnes Press, Jerusalem 1961, and North-Holland Publishing Company, Amsterdam 1962, pp. 328–351.

This is a series of twenty-nine musings about foundational problems. Most of the judgments are wise, but not new. There are also occasional hit-and-run remarks which sacrifice clarity and rational argument for philosophical éclat.

The first nine sections are devoted to various aspects of mechanical mathematics, with some emphasis being placed upon a requirement that proofs be "surveyable." Otherwise, even if a machine claims to have solved Fermat's conjecture in one million steps, we might still not believe it.

The rest of the paper covers so many topics in such short space that no one subject could be adequately developed. Some of the topics are: reduction of mathematics to logic (and set theory), contradictions, mathematical existence.

As samples of hit-and-run assertions, we list the following: (p. 329) "We accept, as a matter of fact, a sequence of symbols as an application of a certain rule, e.g., the modus ponens. Here we may easily get into the slippery ground of truth by convention, synthetic a priori, self-evidence. But an underlying foundation is the sociological fact that it is so accepted"; (p. 346) "It is not necessary to formalize mathematics nor to prove consistency of formal systems if the problem is that bridges shall not collapse unexpectedly. There are many things which are more pertinent in so far as bridges are concerned"; (p. 346) "No formal system which is widely used today is under very serious suspicion of inconsistency" (Quine's NF?); (p. 346) "Admittedly Cantor's well-known definition of the term 'set' is difficult, yet it cannot be denied that the definition does exclude, through the mildly 'genetic' element, the familiar derivation of contradictions"; (p. 349) "... the existence of consistent systems which have no standard models (e.g., are omega-inconsistent) points to a certain discrepancy between existence and consistency ... we can add unnatural numbers to the natural numbers without violating the axioms ... One might argue with reason that although these unnatural numbers are required by the axioms of a consistent system, they should not be said to exist. Such a position would foil the unqualified identification of consistency with existence."

ELLIOTT MENDELSON

RULON WELLS. *A measure of subjective information.* ***Structure of language and its mathematical aspects,*** Proceedings of symposia in applied mathematics, vol. 12, American Mathematical Society, Providence, Rhode Island, 1961, pp. 237–244.

J. D. SABLE, R. WELLS. *Comments.* Ibid., pp. 267–268.

There are concepts of "subjective information" according to which not all *a priori* truths are alike in informativeness; thus, '7+5=12' is less informative than the Pythagorean theorem, and mathematical geniuses are distinguishable by the fact that they make discoveries of enormous informativeness. Rather than dismiss such notions as "merely psychological," it would be useful to have available some good theories about the matter. A *pragmatic* measure of subjective information might be based on the opinion of a battery of experts, care being exercised in various ways in order to render the measure as stable as possible. Such data might be used to found an abstract pragmatic theory with an empirical interpretation.

It might also be possible to construct a purely *semantical* theory of subjective informativeness, using the fiction of an "ideal believer" as a heuretic in defining a measure of the informativeness of sentences which would discriminate among logical equivalents. In such a theory one would have as theorems formal renderings of the following: every conjunction is at least as informative as its conjuncts; sometimes a logical consequence of a sentence is more informative than the sentence itself; some logical equivalents differ in informativeness; some instances of the excluded middle differ in informativeness. Evidently the state-description approach of Carnap and Bar-Hillel will not do for such a concept. It is suggested that its theory might be based on a stronger notion of intensionality such as that of XVII 133 or XXIII 457.

In response to a question raised by Sable, Wells replies that the informativeness of necessarily true sentences should always be less than that of contingent statements, and that only the latter should depend in any way on degree of confirmation.

NUEL D. BELNAP, Jr.

BOLESŁAW SOBOCIŃSKI. *On the single axioms of protothetic.* ***Notre Dame journal of formal logic,*** vol. 1 (1960), pp. 52–73, and vol. 2 (1961), pp. 111–126, 129–148.

These are the first three instalments in a series which is still to be continued. The first article, which is much the most interesting of the three, contains a detailed description, with an abundance of illustrative examples, of how formulae in Leśniewski's protothetic are constructed, and proofs of those obtained; a preliminary metatheorem about the conditions of completeness of a system of protothetic; and an outline history of the successive shortenings of its axiom. The second and third articles contain proofs of the sufficiency of the various axioms tested, with many auxiliary metatheorems.

One point which perhaps might have been made more explicit is that when systems of the sort considered are called "complete," what is meant is quite simply that with respect to any proposition formulable in the system, either that proposition is provable or its negation is. Such completeness is possible because (a) the theses occurring in a system of protothetic have no free variables (they are complete sentences, with all necessary quantifiers, not open sentences or schemata), and (b) the systems are two-valued and extensional.

Despite this last point, the systems are rich enough to be interesting, since they have rules enabling us to introduce constants and variables of all logical types that have propositions as their basis. (Names, and hierarchies involving names, belong elsewhere.) Apart from the types already present in the initial axiom or axioms, variables of a given type can only be used after a constant of the appropriate type has been introduced by a definition, but the rules of definition make this possible with all the types involved in the propositional hierarchy. This is true, at least, of most of the systems involved; there is an introductory discussion of a "restricted" system containing (apart from quantifiers) only propositions and proposition-forming functions of propositional arguments.

These articles, besides containing one or two minor results not previously published, fill a real gap in the literature, at least in English. Mere allusions to procedure and results in this discipline are here replaced by details and by proofs. The account is clearly more faithful to Leśniewski himself than that in Słupecki's XXI 188, though