



Twenty-Third Annual Meeting of the Association for Symbolic Logic

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TWENTY-THIRD ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The twenty-third annual meeting of the Association for Symbolic Logic was held on Thursday, January 22, 1959 at the University of Pennsylvania in Philadelphia in conjunction with the annual meetings of the American Mathematical Society and the Mathematical Association of America.

Professor J. Barkley Rosser of Cornell University delivered an invited address in the afternoon on *Results on the completeness of infinite-valued logics*. Professor Th. Skolem of the University of Oslo was to have delivered an invited address at the morning session on *Set theory based on many-valued logic*, but was unable to do so because of illness.

Six twenty-minute contributed papers were delivered at the morning session which was presided over by Professor Frederic B. Fitch. Four twenty-minute papers were delivered following Professor Rosser's address in the afternoon. The entire afternoon session was presided over by Professor William Craig. The remaining papers were delivered by title.

The Council of the Association met at lunch.

R. M. MARTIN

HASKELL B. CURRY. *The deduction theorem in the combinatory theory of restricted generality.*

The theory referred to is the system \mathcal{F}_2 , described in § 8D of Curry and Feys *Combinatory Logic* 1958 (here cited as [CLg]; see it for notation, references, etc.) The notation $\mathcal{X} \supset_x \mathcal{Y}$ is here used for $\Xi([x]\mathcal{X})([x]\mathcal{Y})$. The deduction theorem for \mathcal{F}_2 is to the effect that if $\vdash \mathcal{X}$ is deducible from the premises $\vdash \xi_k x_1 x_2 \dots x_k$, $k = 1, 2, \dots, m$, then

$$\vdash \xi_1 x_1 \supset_{x_1} \cdot \xi_2 x_1 x_2 \supset_{x_2} \cdot \dots \cdot \xi_m x_1 x_2 \dots x_m \supset_{x_m} ([x_1, \dots, x_m]\mathcal{X})$$

(here abbreviated $\vdash^* \mathcal{X}$) is deducible absolutely. If this were true for unrestricted ξ_i the system would be inconsistent. In [ACT] (see [CLg], p. 385) a formulation of the theory was given in terms of two axiom schemes $[\Xi K]$ and $[\Xi S]$; also it was claimed that the theory is consistent if the values of the Greek letters are restricted to certain "canonical obs", and that the deduction theorem holds for $m=1$ with substitutions for the free variables in the axiom schemes limited to canonical obs. It is now known that, under the same canonicalness restrictions, the general case of the theorem holds if the axiom schemes can be extended by a process of "canonical generalization", i.e. replacement of a free variable α by $\alpha x_1 \dots x_m$ and application of the transformation from $\vdash \mathcal{X}$ to $\vdash^* \mathcal{X}$ (for unspecified ξ_i and m).

G. KREISEL. *The non-derivability of $\neg(x)A(x) \rightarrow (Ex)\neg A(x)$, A primitive recursive, in intuitionistic formal systems.*

We consider Heyting's arithmetic HA (cf. the author's article, *Interpretation of analysis ...* (IN) in *Constructivity in Mathematics*, 1958, North-Holland, para. 1.2) and Kleene's formalization of intuitionistic analysis (IA) (mimeographed Summaries of talks presented at the Summer Institute of Symbolic Logic in 1957 at Cornell University, pp. 100-104). The interpretation of IN, para. 3.52, shows that $\neg(x)A(x) \rightarrow (Ex)\neg A(x)$ is provable in the systems considered only if $(Ey)[\neg(x)A(x) \rightarrow \neg A(y)]$, hence $(x)A(x) \vee (Ey)\neg A(y)$, is provable. More generally, if \mathcal{A} and $\mathcal{B}(x)$ do not contain existence or disjunction symbols, and $\mathcal{A} \rightarrow (\exists x)\mathcal{B}(x)$ is provable in the systems considered, then $(Ex)[\mathcal{A} \rightarrow \mathcal{B}(x)]$ is also provable. (In the case of HA it is necessary to eliminate the functionals of para. 3.1, which is done by interpreting them as effective operations (e.o.) of para. 4.2 and proving in HA the existence of e.o. satisfying the axioms of para. 3.1.) If $A(x)$ contains no variables other than x , $(x)A(x) \vee (Ex)\neg A(x)$

is only provable if one of the alternatives is provable, and so, by Gödel's construction of formally undecidable formulae $(x)A(x)$ with primitive recursive A , each of the systems considered contains A for which $\neg(x)A(x) \rightarrow (Ex)\neg A(x)$ is not provable. *Corollary.* Consider Heyting's predicate calculus (HPC) and define strong completeness as follows; for all formulae \mathfrak{A} of HPC, if P_1, \dots, P_k are the predicate symbols of $\mathfrak{A}(P_1, \dots, P_k)$, and if, for all domains of individuals and predicates P_i^* defined there, $\mathfrak{A}(P_1^*, \dots, P_k^*)$ holds then the formula $\mathfrak{A}(P_1, \dots, P_k)$ is provable in HPC. Gödel observed (personal communication) that if strong completeness of HPC is intuitionistically provable, then so is $\neg(x)A(x) \rightarrow (Ex)\neg A(x)$ for each primitive recursive A . Thus, strong completeness of HPC is not provable in HA, and since (IA) contains the fan theorem it is not even provable by means of the latter. Note that our non-derivability result for HA cannot be obtained by means of the (classical) realizability interpretation nor by Gödel's interpretation (IN, para. 3) since the implication is true for each primitive recursive A on both interpretations.

ROBERT McNAUGHTON. *Logical problems in the description of the behavior of automata.*

The behavior of an automaton, as opposed to its structure, deals with the relation between inputs and outputs; more precisely, the exact manner in which the inputs determine the outputs. In this talk I investigate the capability of several languages for describing the behavior of automata. Some of these languages are in the form of symbolic logic; others are not. Some theorems are proved about the adequacy of these languages, and problems for future logical research are pointed out.

ALAN ROSS ANDERSON and NUEL D. BELNAP, JR. *A modification of Ackermann's "rigorous implication".*

Following a suggestion of Feys, we thus translate Ackermann's *strenge Implikation* (XXII 327). We consider three systems, all of which we take to be contributions to the problem of formalizing *entailment*: Ackermann's system Π' , consisting of his axioms (1)–(15), rules (α) – (δ) ; Ackermann's Π'' , got by adding axioms (16)–(17) and rule (ϵ) to Π' ; and a new system E (for "entailment"), consisting of axioms (2)–(15) together with

$$(1') \quad (((A \rightarrow A) \ \& \ (B \rightarrow B)) \rightarrow C) \rightarrow C,$$

and rules (α) (*modus ponens* for \rightarrow), and (β) (adjunction).

(I) Two remarks of Ackermann require correction. A theory of modality equivalent to that of Π'' is available in Π' , the required definition of impossibility UA being $A \rightarrow \overline{A \rightarrow A}$; hence (16)–(17) and (ϵ) are unnecessary. And $MMA \rightarrow MA$ is provable in all three systems. So also in fact is $M \sim MA \leftrightarrow M \sim M \sim M \sim MA$, whence it follows by an argument of Parry (V 37) that the three systems distinguish just those fourteen modalities distinguished by S4. (II) None of the three systems contains or is contained in any of the following: S1, S2, S3, M (Feys-von Wright), but all three are contained in S4. Adding $B \rightarrow (A \rightarrow A)$ to any of the systems yields S4. (III) E contains the full two-valued system (with $A \supset B$ defined as $\overline{A \vee B}$), and has as derived rules (δ) and a rule of necessitation. E has the same theorems as Π' without (γ) (though adding (γ) to E does not yield Π'). (IV) Define *proof that A entails B* as a proof of B from hypothesis A which satisfies the following conditions: (a) stars can be prefixed to steps of the proof as required by the following rules: (i) the hypothesis is starred; (ii) a step introduced as an axiom is not starred; (iii) the conclusion of an application of (α) is starred if and only if at least one premiss is starred; (iv) the conclusion of an application of (β) is starred if both premisses are starred, and unstarred if both premisses are unstarred (and no use of (β) has one premiss starred and the other unstarred);

and (b), in consequence of the rules of (a), \mathbf{B} has a star. (The conditions are designed to explicate what we might mean by saying that \mathbf{B} "really depends on" \mathbf{A} (Ackermann's *logischer Zusammenhang*.) Our principal interest in \mathbf{E} (as opposed to \mathbf{II}' and \mathbf{II}'') is that for \mathbf{E} we have the following: *Entailment theorem*. $\vdash \mathbf{A} \rightarrow \mathbf{B}$ if and only if there is a proof that \mathbf{A} entails \mathbf{B} .

J. W. ADDISON. *Some consequences of the axiom of constructibility*. (The abstract has already appeared in *Notices of the American Mathematical Society*, Vol. 5, December 1958, p. 845.)

RAYMOND M. SMULLYAN. *Theories with effectively inseparable nuclei*.

We shall refer to the set of Gödel numbers of the provable sentences and the set of Gödel numbers of the refutable sentences of a theory T as the *nuclei* of T . We call T a *Rosser theory* iff every disjoint pair (A, B) of r.e. sets is *strongly separable* in the theory, in the sense that there exists a formula $F(x)$ such that for every $n \in A$, $F(n)$ is provable, and for every $n \in B$, $F(n)$ is refutable in T . It is easy to show that the nuclei of a Rosser theory are recursively inseparable — in fact effectively inseparable; a fortiori every Rosser theory is essentially creative (i.e. all its consistent extensions are creative). In fact we easily prove the stronger theorem "Let (A, B) be strongly separable in T . Then (1) If (A, B) are recursively inseparable, so are the nuclei of T ; (2) If (A, B) are effectively inseparable, so are the nuclei of T ." [This theorem is a strengthening of Kleene's symmetric form of Gödel's theorem; it is also a strengthening of a result of Feferman — this JOURNAL, vol. 22 (1957), page 170, Corollary 3.] We show that if T_1 is interpretable or relatively interpretable in T_2 then: (1) If T_1 has recursively inseparable nuclei, so does T_2 ; (2) Likewise with "effectively inseparable." [Two sets α and β are called effectively inseparable iff there exists a recursive function $f(x, y)$ such that for any two numbers i and j which are indices of disjoint r.e. supersets ω_i and ω_j of α and β , $f(i, j)$ is outside both ω_i and ω_j .] By our methods, many theories shown by Tarski to be essentially undecidable, and subsequently shown by Feferman to be essentially creative, can now be shown to have effectively inseparable nuclei. In particular, the elementary theories of non-densely ordered rings and non-densely ordered commutative rings (with or without unit) have effectively inseparable nuclei.

ANDRZEJ MOSTOWSKI and CZESŁAW RYLL-NARDZEWSKI. *Representability of sets in models of axiomatic theories*.

Let S be a consistent axiomatizable theory with standard formalization and $\{F_n(x)\}$ a recursive sequence of formulas of S with one free variable x . For any model M of S and any a in M write $Sisf_M F_n(a)$ if a satisfies $F_n(x)$ in M . Call a set X of integers *representable* in M by a with respect to the sequence $\{F_n(x)\}$ if $n \in X \equiv_n Sisf_M F_n(a)$. THEOREM 1: *For every sequence X_j of non-recursive sets $\{X_j\}$ there is a model M of S in which none of the sets X_j is representable by any a in M with respect to any recursive sequence $\{F_n(x)\}$* . This theorem is proved by considering the space \mathfrak{S} consisting of prime ideals of the Lindenbaum algebra of S and by showing that those ideals which determine models with the required property form a residual G_δ -set in \mathfrak{S} . — Let G_n be a recursive sequence of closed formulas. Call a set X of integers *definable* in S with respect to the sequence $\{G_n\}$ if $n \in X \equiv_n [G_n \text{ is provable in } S]$. From theorem 1 one obtains immediately THEOREM 2: *For every sequence $\{X_j\}$ of non-recursive sets there is a consistent extension S' of S such that none of the sets X_j is definable in S' with respect to any recursive sequence $\{G_n\}$* . Theorem 1 was proved by the first author for a single set X_j , a particular system S and a particular sequence $\{F_n(x)\}$; the generalization and the proof using topological methods were obtained by the second author.

DANIEL LACOMBE. *A convenient definition for functionals.*

Let F_p be the set of all functions of p non-negative integers with non-negative integers values, PF_p the set of all partial functions of p variables, FF_p the set of all partial functions of p variables with finite domains of definition. Using certain subsets of $FF_a \times \dots \times FF_l \times FF_m$ (where possibly some of the a, \dots, l, m are null) we define: (A) the pseudo-functionals (which involve possible contradictions); (B) the functionals (which are the same as the consistent pseudo-functionals, and are continuous mappings of $PF_a \times \dots \times PF_l$ into PF_m); (C) the totally functionalizing functionals (which induce continuous mappings of $F_a \times \dots \times F_l$ into F_m); (A'), (B'), (C') respectively obtained from (A), (B), (C) by adding a recursivity condition; and (D) the restricted recursive functionals (among which are most of the usual recursive functionals, for instance the μ -schema). (A') [(B')] are equivalent to partial recursive [everywhere defined] functionals in the sense of KLEENE. We study these different classes and their applications, and give a normal form for (B'). This method is easily extended to many other topological spaces. Proceeding by induction on the type we can obtain the countable functionals (which correspond to (C)) and the recursively countable functionals (which correspond to (C')) as defined by KLEENE and KREISEL; but for these higher types the notion of countability thus defined is not identical with the notion of continuity.

MARIAN BOYKAN POUR-EL. *Computable functionals.*

In the search for an adequate definition of the intuitive notion "computable functional" the definitions of *partial recursive functional* and *effective operation* have occupied a prominent place. An open question concerning the relationship between these two definitions is whether every effective operation is a partial recursive functional. We answer this question in the negative.

THEOREM I. There exist effective operations which are not partial recursive functionals.

The original proof was obtained by topologizing the space of number-theoretic functions and exhibiting an effective operation which is discontinuous at certain points of its domain. Since a partial recursive functional is continuous at all points of its domain the theorem follows. [Myhill obtained such an example by topological considerations independently of the author.] A study was made of the discontinuities and fixed points of such effective operations. It was then noticed that topological arguments are not necessary and a proof of theorem I was obtained directly from the definitions.

As a consequence of theorem I we have

THEOREM II. There are two families of functions F and G such that F and G are recursively separable but not topologically separated.

In spite of theorem I, the following continuity results hold.

THEOREM III. If 1) Φ is a Banach-Mazur functional such that a) the domain of $\Phi(\delta\Phi)$ is a subset of the partial recursive functions, b) the range of $\Phi(\rho\Phi)$ is a subset of the (general) recursive functions,

2) F is a recursively enumerable family of functions such that $F \subset \delta\Phi$,

3) p is a recursive function such that $p \varepsilon \delta\Phi$,

then Φ is continuous at p on the topological space $F \cup \{p\}$.

THEOREM IV. If 1) Φ is as in 1) of theorem III

2) F is a recursively enumerable family of (general) recursive functions such that $F \subset \delta\Phi$,

3) F^* consists of F together with all those general recursive limit points $p \varepsilon \delta\Phi$, then Φ is continuous with respect to the topology induced on F^* at every point $p \varepsilon F^*$.

Since effective operations and partial-recursive functionals are Banach-Mazur,

theorems III and IV hold for these functionals. By specializing these theorems several applications have been obtained.

RICHARD MONTAGUE. *The continuum of relative interpretability types.*

It has been found possible to extend the results concerning relative interpretability which were announced in the *Notices of the American Mathematical Society*, vol. 6 (1958), pp. 362-3. As in Tarski, Mostowski, Robinson, *Undecidable theories*, \mathbf{N} is the theory whose valid sentences are the truths of the elementary arithmetic of natural numbers, and \mathbf{Q} is a certain finitely axiomatizable subtheory of \mathbf{N} . If ϕ is any expression, then $\bar{\phi}$ is the numeral of \mathbf{Q} corresponding to the Gödel number of ϕ . *Lemma.* If ψ is a formula of \mathbf{Q} whose free variables are v_0, \dots, v_n , then there are infinitely many formulas ϕ of \mathbf{Q} such that the free variables of ϕ are v_0, \dots, v_{n-1} and $\vdash_{\mathbf{Q}} \phi(v_0, \dots, v_{n-1}) \leftrightarrow \psi(v_0, \dots, v_{n-1}, \bar{\phi})$.

Theorem. If ϕ is any valid sentence of \mathbf{N} , then there is an infinite set A of valid sentences of \mathbf{N} such that each member of A logically implies ϕ , and no member ψ of A is relatively interpretable in the theory axiomatized by the members of A other than ψ .

Definition. The *relative interpretability type* of a theory T is the class of all theories which are relatively interpretable in T and in which T is relatively interpretable.

Corollary. The class of relative interpretability types of subtheories of \mathbf{N} has the power of the continuum.

G. KREISEL. *The Cantor-Bendixson theorem in hyperarithmetic analysis.*

The following basic notation will be used: constants 0 and ' (successor), =, individual variables n, m, \dots (intended to range over natural numbers), a, b, \dots (over rationals), $\alpha, \beta, \gamma, \dots$ over functions of one or more arguments: we are concerned with truth, not provability. A large body of statements of informal analysis can be expressed in the obvious way in the above notation when real numbers ξ are represented by $\alpha, \alpha(n) = 0 \vee \alpha(n) = 0'$ (corresponding to $\xi = \sum \alpha(n)2^{-n}$), sequences ξ_m by $\alpha(m, n)$, open sets G on the line by the end points (a_n, b_n) of a sequence of rational intervals of which G is composed, closed sets F by the complementary open set. A statement A is called *true in a class M* of functions of natural numbers if A is true when the function variables occurring in A are restricted to range over M . We shall be especially interested in $M = K_d, K_d$ the set of hyperarithmetic (h.a) functions of degree d and $d \varepsilon O$ (the set of recursive ordinal notations), and $M = K^*, K^* = \cup K_d$. If $|d|$ is a limit number, and $a_n, b_n \varepsilon K_d$, the following properties (i), (ii) are invariant for all $K \supset K_d$: (i) if a, b are rational, $F \cap [a, b]$ contains at least 1 (2) point; for if $[a, b]$ (for some $c, d, a < c < d < b$, both $[a, c]$ and $[d, b]$) cannot be covered by finitely many (a_n, b_n) then there is at least 1 (2) point of degree $|d|$ in $F \cap [a, b]$, and (ii) F is perfect; for F is perfect if and only if, for all a, b , if $F \cap [a, b]$ contains 1 point, it contains 2. The Cantor-Bendixson theorem (CB) states: any closed set is the union of a perfect set and a sequence of points. *Theorem.* CB is false when relativized to $K_d, |d|$ limit number, or to K^* . In fact, a certain recursively closed set independent of $|d|$ provides a counter example. *Proof.* There is a primitive recursive $R(n, m)$ such that for any $e \varepsilon O$ there is a unique α satisfying $(x)R[e, \bar{\alpha}(x)]$ and this α is of degree e . We map the set of all β recursively into the non-terminating binaries of $I_e = (2^{-e}, 2^{-e} + 2^{-(e+2)})$; then $\hat{\beta}(Ex) \rightarrow R[e, \bar{\beta}(x)]$ maps into the complement, with respect to $\{2^{-e}\} \cup I_e$, of a recursively closed, scattered set $J_e. \{0\} \cup \Sigma J_e$ is recursively closed, and for $e \varepsilon O$ contains a real number of degree $|e|$ in its scattered (non-perfect) part. The latter cannot be enumerated by a sequence of degree $|d|$; for if $|d|$ is a limit number, a sequence of degree $|d|$ is of degree $|d_1|, |d_1| < |d|$, and this cannot enumerate all elements of J_e , for $|e| = |d_1| + 1$, though $|d_1| + 1 < |d|$. *Corollary.* Call the statement \mathfrak{A} , namely $(\alpha_1)(E\beta_1), \dots, (\alpha_n)(E\beta_n)A(\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_n)$, h.a. realizable if

there are h.a. functionals $B_i(\alpha_1, \dots, \alpha_i)$ such that $A[\alpha_1, \dots, \alpha_n, B_1(\alpha_1), \dots, B_n(\alpha_1, \dots, \alpha_n)]$ for all α : CB is not h.a. realizable. *Definition.* \mathfrak{A} is called *stable* (with respect to h.a. closure) if there is a $d, d \in 0$, with the property: for each $\alpha_1^* \in K_d$, there is a $\beta_1^* \in K_d \dots$ for each $\alpha_r^* \in K_d$ there is a $\beta_r^* \in K_d$ such that $(\alpha_{r+1})(E\beta_{r+2}) \dots (\alpha_n)(E\beta_n) A(\alpha_1^*, \dots, \alpha_r^*, \alpha_{r+1}, \dots, \alpha_n, \beta_1^*, \dots, \beta_r^*, \beta_{r+1}, \dots, \beta_n)$ is true in M provided only M contains ' and is closed under h.a. operations of degree $|d|$. H.a. realizable theorems are stable. The converse is open. It is proposed to characterize *predicative theorems* by this property of stability (relative to the identification of predicative operations with h.a. ones): the theorem remains true (for β_i^* determined by α_j^* , $j \leq i \leq n$) when the range M of the bound function variables varies, subject only to a predicative closure condition. In other words, the precise extent of such M does not affect the truth of the theorem: this is the essential difference to impredicative theorems.

MAKOTO ITOH. *The n-valued logics and the lattices of n-valued functions.*

The ordinary two-valued logic is regarded mathematically as a Boolean lattice and since each Boolean lattice can, as was shown by Stone, be represented by a lattice of two-valued functions on a set R (which is equivalent to the lattice of subsets of R), we may consider such a lattice of two-valued functions as a mathematical representation of the ordinary two-valued logic. Extending this view point to the case of many-valued logics, we shall here first find the characteristic properties of a lattice of n -valued functions on a set R and then induce a complete set of axioms for this lattice which may be considered at the same time to represent the axioms for an n -valued logic, too.

MAKOTO ITOH. *General solution of a general n-valued logical equation.*

By making use of the theorems concerning the n -valued logics which have been obtained in my other paper, we shall give here the general solution of a general n -valued logical equation with one or many unknown elements, together with the necessary and sufficient condition in order that the solution do exist.

If we take specially $n = 2$, then we get the general solution of a general Boolean equation as a corollary of the above result.

The ASSOCIATION FOR SYMBOLIC LOGIC announces the following elections each for a period of three years from January 1, 1959.

For President, Professor Frederick B. Fitch, of Yale University, New Haven, Connecticut.

For Vice-President, Dr. William Craig, of The Pennsylvania State University, University Park, Pennsylvania.

As members of the Executive Committee Professor Henry Mehlberg, of University of Chicago, Chicago, Illinois, and Dr. Robert L. Vaught, of University of Washington, Seattle, Washington.

As members of the Council, Professor J. Dopp, of Université de Louvain, Louvain, Belgium, and Professor Roland Fraïssé, of Université d'Alger, Algiers, Algeria.

There will be a meeting of the ASSOCIATION FOR SYMBOLIC LOGIC at Columbia University, New York, on Monday, December 28, 1959, in conjunction with a meeting of the American Philosophical Association. Members desiring to submit abstracts will please do so in duplicate by November 1, 1959, sending them to the chairman of the Program Committee, Prof. Alan Ross Anderson, Department of Philosophy, Yale University, New Haven, Connecticut.