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ENTAILMENT AND RELEVANCE¹

NUEL D. BELNAP, JR.

Those who object to the identification of strict implication and entailment speak in such a way that for A to entail B , A must be *relevant* to B . This view of entailment agrees not only with that of the naive student of logic (who tends to think the phrase *paradoxes of "implication"* more accurate than "*paradoxes of implication*"), but also with that expressed in the initial chapters of many textbooks in logic under the heading "Fallacies of Relevance."²

In regard to formal theories proposed as explications of the notion of entailment, it becomes important to have at our disposal some *formal* account of the demand for relevance as between antecedent and consequent of an entailment. One such account is that given by Anderson in [2], where he explicates the notion of "depends on the logical content of" in terms of the actual use of a premiss in getting to a conclusion, demonstrating that in the system E of [3], $A \rightarrow B$ is provable if and only if B "depends on the logical content of" A .

An alternative account is suggested by informal remarks of some of the critics of strict implication. Thus, Nelson says ([6], p. 445) that implication [entailment] "is a necessary connection between meanings," Duncan-Jones ([5], p. 71) that A entails B only when B "arises out of the meaning of" A , and Baylis ([4], p. 397) that entailment obtains only when "the intensional meaning of B is identical with a part of the intensional meaning of A ." Evidently these writers agree that in a formal theory of entailment, we would wish to reject $A \rightarrow B$ if A and B have nothing to do with one another, if, that is, A and B are totally disparate in meaning. The problem is to restate this condition in formal terms.

Confining our attention to propositional logic, a partial solution becomes almost obvious once we notice that in propositional logic, commonality of meaning is carried by identity of propositional variables. Thus, for A to be relevant to B in the required sense, a necessary condition is that A and B have some propositional variable in common. (It is the absence of this feature, I think, that makes $A \wedge \bar{A} \rightarrow B$ seem "outrageous" to Duncan-Jones and "utterly devoid of rationality" to Nelson.)

It can be shown that the system E of [3] satisfies this condition; i.e.,

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² Regard for the importance of relevance as a criterion of validity is generally suppressed in the later, more formal chapters of such textbooks.

$A \rightarrow B$ is provable in E only if A and B share a variable.³ Consider the matrices below.⁴

Every axiom of E takes a designated (+) value under every assignment of values to its variables; and the rules of E preserve this property.⁵

If $A \rightarrow B$ is such that A and B have no variable in common, then there is an assignment of values to the variables of $A \rightarrow B$ such that $A \rightarrow B$ takes an undesigned value: to wit, let the variables of A each take the value +1 and the variables of B each take the value +2. Then A will take a value ± 1 and B a value ± 2 ; but $\pm 1 \rightarrow \pm 2$ takes the undesigned value -3. Hence if $A \rightarrow B$ is a theorem of E, A and B share a variable.

$A \rightarrow B$													
-3	-2	-1	-0	+0	+1	+2	+3		\bar{A}	NA	MA		
-3	+3	+3	+3	+3	+3	+3	+3	+3	-3	+3	-3	-3	
-2	-3	+2	-3	+2	-3	-3	+2	+3	-2	+2	-2	-2	
-1	-3	-3	+1	+1	-3	+1	-3	+3	-1	+1	-1	-1	
-0	-3	-3	-3	+0	-3	-3	-3	+3	-0	+0	-0	-0	
+0	-3	-2	-1	-0	+0	+1	+2	+3	+0	-0	+0	+0	
+1	-3	-3	-1	-1	-3	+1	-3	+3	+1	-1	+1	+1	
+2	-3	-2	-3	-2	-3	-3	+2	+3	+2	-2	+2	+2	
+3	-3	-3	-3	-3	-3	-3	-3	+3	+3	-3	+3	+3	

$A \wedge B$									$A \vee B$								
-3	-2	-1	-0	+0	+1	+2	+3		-3	-2	-1	-0	+0	+1	+2	+3	
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-2	-1	-0	+0	+1	+2	+3
-2	-3	-2	-3	-2	-3	-3	-2	-2	-2	-2	-2	-0	-0	+2	+3	+2	+3
-1	-3	-3	-1	-1	-3	-1	-3	-1	-1	-1	-0	-1	-0	+1	+1	+3	+3
-0	-3	-2	-1	-0	-3	-1	-2	-0	-0	-0	-0	-0	-0	+3	+3	+3	+3
+0	-3	-3	-3	-3	+0	+0	+0	+0	+0	+0	+2	+1	+3	+0	+1	+2	+3
+1	-3	-3	-1	-1	+0	+1	+0	+1	+1	+1	+3	+1	+3	+1	+1	+3	+3
+2	-3	-2	-3	-2	+0	+0	+2	+2	+2	+2	+2	+3	+3	+2	+3	+2	+3
+3	-3	-2	-1	-0	+0	+1	+2	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3

Designated values: +0, +1, +2, +3.

³ This property is independent of that demonstrated by Ackermann [1] for his system Π' (which includes E), according to which one does not have $A \rightarrow (B \rightarrow C)$ as a theorem unless A contains some wf part ($D \rightarrow E$). The present matrices also suffice to show that Ackermann's system Π' has the required property of relevance.

⁴ I have included matrices for $NA = \text{df } (A \rightarrow A) \rightarrow A$ and $MA = \text{df } \overline{N\bar{A}}$.

⁵ I am indebted to John Wallace and David Levin for the detailed verification of this fact.

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