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Review: [untitled]

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non-logical constants occurring in his axioms are 'C', which denotes the confirmation function; the symbol '⊢', which expresses the property "is a logically true sentence"; symbols denoting the familiar sentential operations; and various mathematical symbols. Leblanc, in his consistency proofs, assigns interpretations, not only to 'C' but also to '⊢', under which the axioms become true (giving other symbols their usual sense). The interesting consistency question, however, is not whether such interpretations can be given but whether, for each of the axiom systems, there is an interpretation of 'C' such that if all the other symbols, including '⊢', receive their usual sense, the axioms will be true. This second sense of consistency is indeed suggested by the author's practice; in deriving consequences from his axioms, he frequently employs properties of '⊢' which are not explicit consequences of the axioms.

With one exception, the various axiom systems proposed by Leblanc can very easily be shown to be consistent in this second sense (though not, of course, by Leblanc's arguments). The exception is the system due to Kemeny, composed of A1-A5, which is inconsistent in view of the argument in the second paragraph of Leblanc's paper.

Author's correction: Replace axiom B3 (p. 348) by 'If  $\vdash h$ , then  $C_0(h) = 1$ .'

RICHARD MONTAGUE

A. N. PRIOR. *Peirce's axioms for propositional calculus*. *The journal of symbolic logic*, vol. 23 no. 2 (for 1958, pub. 1959), pp. 135-136.

The following axioms due to Peirce are considered: 1.  $C\dot{p}\dot{p}$ ; 2.  $CC\dot{p}Cq\dot{r}CqC\dot{p}\dot{r}$ ; 3.  $CC\dot{p}qCC\dot{q}\dot{r}C\dot{p}\dot{r}$ ; 4.  $Cop$ ; 5.  $CCC\dot{p}q\dot{p}\dot{p}$ . In 4 the proposition  $o$  is to be viewed as an arbitrary false proposition. Using modus ponens and substitution, Peirce derived the following theorems from the above axioms: 6.  $C\dot{p}CC\dot{p}q\dot{q}$ ; 7.  $C\dot{p}Cq\dot{p}$ ; 8.  $CNN\dot{p}\dot{p}$ . Here  $N\dot{p}$  is defined as  $C\dot{p}o$ .

The above basis can be seen to be sufficient for the propositional calculus because Wajsberg has shown that 3, 4, 5, and 6 constitute a sufficient basis for the propositional calculus, and because Peirce derived 6 from 1 and 2. Prior gives a detailed derivation of 8. He uses matrices to establish the independence of 2, 3, 4, and 5, and he outlines a proof of 1 from 2, 3, 4, and 5.

FREDERIC B. FITCH

HUGUES LEBLANC and THEODORE HAILPERIN. *Nondesignating singular terms*. *The philosophical review*, vol. 68 (1959), pp. 239-243.

The writers address themselves to the problem of existential generalization on singular terms which have no referent — e.g., "Pegasus." They remark that the following proposals are unsatisfactory: (1) altogether doing without non-designating individual constants is "awkward"; (2) replacing all individual constants by definite descriptions (Quine) is, in terms of the complexity of the machinery introduced, "rather costly," as is (3) Leonard's proposal (*Philosophical studies*, vol. 7 (1956), 49-64) to solve the problem by introducing modal operators and quantification over predicates.

They therefore offer new rules of singular inference based on a sharp distinction between individual variables and individual constants. Let  $x$  and  $x'$  range over individual variables,  $t$  over individual constants, and  $a$  and  $a'$  over both. Then in addition to standard rules for the propositional connectives, Leblanc and Hailperin propose as rules for identity and existential quantification the following: IE\*.  $a = a'$ ,  $Aa \vdash Aa'$ . II\*.  $\vdash a = a$ .  $\exists E$ . If  $A_1, \dots, A_n, Bx' \vdash C$  and  $x'$  is bound except in  $Bx'$ , then  $A_1, \dots, A_n, (\exists x)Bx \vdash C$ .  $\exists I^*$ .  $Ax' \vdash (\exists x)Ax$ . From these we have:  $\exists I^{**}$ .  $(\exists x)(t = x)$ ,  $At \vdash (\exists x)Ax$ , but not the proscribed  $At \vdash (\exists x)Ax$ . The writers remark that weakening IE\* and II\* by restricting  $a$  to variables, and weakening  $\exists I^*$  to  $\exists I^{**}$ , leads to an unsatisfactorily weak theory of identity, since

then one would not have  $t = t$ ; and they defend the view that  $t = t$  is true regardless of whether or not  $t$  designates. (See reviewer's comment at end of review below.)

NUEL D. BELNAP, JR.

JAAKKO HINTIKKA. *Existential presuppositions and existential commitments*. *The Journal of Philosophy*, vol. 56 (1959), pp. 125–137.

Hintikka relates the problem of singular terms which do not or might not designate (e.g. *Homer*, *N. Bourbaki*) to Quine's thesis (QT), "To be is to be a value of a bound variable." He argues that resorting to Russell's theory of definite descriptions in order to defend QT's applicability to non-designating singular terms is unsatisfactory, and prefers rather to modify the underlying quantificational logic. His modification is essentially the same as that of the paper reviewed above. (The two papers are independent; the reviewer has been informed that they were submitted within five days of one another.)

Hintikka bases his formulation on a metalogical, monotonic equivalence relation  $\leftrightarrow$ , abbreviating  $A \leftrightarrow A \& B$  as  $A \rightarrow B$ . The axioms and rules are such that one does not have  $A \leftrightarrow B$  unless  $A$  and  $B$  contain occurrences of exactly the same free variables; since this feature does not contribute to the main theme of his paper, it is possible he intended to offer a bit of *lagniappe* by formalizing the concept of entailment. It is therefore worth remarking that  $A \& \bar{A} \& \bar{B} \rightarrow B$  is provable, though this seems a dubious entailment. On the other hand, Hintikka's system seems wholly in the spirit of Parry's *analytische Implikation* (473I).

In order to relate existential presuppositions to existential commitments, Hintikka bids us take  $(\exists x)(x = t)$  as a translation of " $t$  is a value of a bound variable," and hence by QT, of " $t$  exists." Given the new system with  $(\exists x)(t = x) \& At \rightarrow (\exists x)Ax$ , but not  $At \rightarrow (\exists x)Ax$ , it follows that QT is equivalent with Quine's "older thesis," that for " $t$  exists" to be true, it is necessary and sufficient that existential generalization on  $t$  be valid.

Since it is not presupposed that the individual constants designate, Hintikka labels his system a "logic without existential presuppositions" (first sense); but it is of course not such in the (second) sense of being valid in the empty domain. A logic which is without existential presuppositions in *both* senses seems to the reviewer more in the spirit of the enterprise. Existing proposals are, however, unsatisfactory for present purposes: Mostowski's (XVI 272(1)) for reasons cited by Hailperin and Quine (XX 284), and the systems of the latter because of the absence of free variables or individual constants, required for applicability to singular terms.

One terminological matter needs clarification: Hintikka uses "free variable" and "bound variable" to distinguish two kinds of expressions of the alphabet, called by Leblanc and Hailperin "individual constant" and "variable" respectively. Thus Hintikka is led to speak somewhat confusingly of "expressions in which all bound variables are actually bound to some quantifier." In the first use of "bound" a kind of expression is distinguished, while in the second use the manner of its occurrence is indicated.

NUEL D. BELNAP, JR.

K. J. J. HINTIKKA. *Towards a theory of definite descriptions*. *Analysis* (Oxford), vol. 19 no. 4 (1959), pp. 79–85.

By building on the "logic without existential presuppositions" of the article reviewed above, Hintikka formulates a theory of definite descriptions which fails to have some of the counterintuitive consequences of Russell's theory for empty descriptions. The new theory is constructed in two steps. (i) Definite descriptions  $(\iota x)Bx$  are admitted "on the same footing with" individual constants, thus assimilating them to ordinary names. Hintikka presumably intends thereby that if  $At$  is a theorem, then