University of Pittsburgh
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Partial Differential Equations 1 – Spring 2019
Exercise Sheet 6 — Due Date: April 24

Work in groups, write in LaTeX!

Problem 21

In the lectures we proved the Poincaré inequality.

Let $\Omega \subset\subset \mathbb{R}^n$ and $\partial\Omega \in C^\infty$ then for any $p \in (1,\infty)$ there exists a constant $C(\Omega, p)$ such that

$$\|u\|_{L^p(\Omega)} \leq C(\Omega, p)\,\|Du\|_{L^p(\Omega)}. \tag{1}$$

holds for any $u \in W^{1,p}_0(\Omega)$.

Show that this implies the following. For any $p \in (1,\infty)$ there exists a constant $C(n, p)$ such that for any ball $B(x_0, r)$ of radius $r > 0$, $x_0 \in \mathbb{R}^n$

$$\|u\|_{L^p(B(x_0,r))} \leq C(n, p)\, r \|Du\|_{L^p(B(x_0,r))}.$$

holds for any $u \in W^{1,p}_0(B(x_0, r))$.

For this you can proceed as follows:

(i) Set $v(x) := u(r(x - x_0))$. Show that $v \in W^{1,p}_0(B(0, 1))$ and conclude from (1) for $\Omega := B(0, 1)$

$$\|v\|_{L^p(B(0,1))} \leq C(n, p)\,\|Dv\|_{L^p(B(0,1))}.$$

(ii) Use the scaling arguments from the last exercise (Problem 20) to conclude.
