1. Let $f, g : [a, b] \to \mathbb{R}$ be Riemann integrable and $\lambda, \mu \in \mathbb{R}$. Show that $\lambda f + \mu g$ is Riemann integrable and we have

$$\int_{[a,b]} (\lambda f + \mu g) = \lambda \int_{[a,b]} f + \mu \int_{[a,b]} g$$

2. Let $f : [a, b] \to \mathbb{R}$ be a bounded function, which is continuous in $[a, b] \backslash \Sigma$. Asume that Σ is a countable set $\Sigma = \{c_1, c_2, \ldots\}$. Without using Riemann-Lebesgue theorem, show that f is Riemann integrable.

3. Let $f : [a, b] \to \mathbb{R}$ be Riemann-integrable and let $g : [a, b] \to \mathbb{R}$ such that $f(x) = g(x)$ for all $x \in [a, b] \backslash \Sigma$ where $\Sigma = \{x_1, \ldots, x_n\}$. Show that then g is Riemann integrable in $[a, b]$ and we have

$$\int_{[a,b]} f = \int_{[a,b]} g.$$

4. (bonus question) Prove the positive part of the Riemann-Lebesgue theorem:

   Let $f : [a, b] \to \mathbb{R}$ be bounded and assume that $f : [a, b] \to \mathbb{R}$ is continuous in $[a, b] \backslash \Sigma$ for some $\Sigma \subset \mathbb{R}$ with Lebesgue measure zero. Show that f is Riemann-integrable.