

1. Complete these problems from the book:

- (a) pg. 109, Exercise 3.2.10
- (b) pg. 115, Exercise 3.3.4, Exercise 3.3.7
- (c) pg. 120, Exercise 3.4.3, Exercise 3.4.4

2. A function  $f : D \subset \mathbb{R} \rightarrow \mathbb{R}$  is called (*sequentially*) *lower semicontinuous* at a point  $x \in D$  if we have

$$f(x) \leq \liminf_{D \ni y \rightarrow x} f(y),$$

in the sense that for any sequence  $(y_n)_{n \in \mathbb{N}} \subset D$  with  $\lim_{n \rightarrow \infty} y_n = x$  we have

$$f(x) \leq \liminf_{n \rightarrow \infty} f(y_n).$$

In a similar spirit, a function is called (*sequentially*) *upper semicontinuous* if

$$f(x) \geq \limsup_{D \ni y \rightarrow x} f(y).$$

- (a) Give an example of a lower semicontinuous function which is not continuous.
- (b) Give an example of an upper semicontinuous function which is not continuous.
- (c) Show that  $f$  is continuous at  $x \in D$  if and only if  $f$  is lower and upper semicontinuous at  $x$ .
- (d) Show that if  $f : [a, b] \rightarrow \mathbb{R}$  is lower semicontinuous in every  $x \in [a, b]$ , then  $f$  attains its minimum value in  $[a, b]$ .
- (e) Show that if  $f : [a, b] \rightarrow \mathbb{R}$  is upper semicontinuous in every  $x \in [a, b]$ , then  $f$  attains its maximum value in  $[a, b]$ .