

1. Complete these problems from the book:

(a) p.75, Exercise 2.4.1

(b) p.75, Exercise 2.4.2

2. Prove the following statement using Bolzano-Weierstrass theorem.

Assume that $(x_n)_{n \in \mathbb{N}}$ is a *bounded* sequence in \mathbb{R} and that there exists $x \in \mathbb{R}$ such that any *convergent* subsequence $(x_{n_i})_{i \in \mathbb{N}}$ converges to x . Then $\lim_{n \rightarrow \infty} x_n = x$.

3. Show that

(a) the set $\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$ has no cluster points.

(b) every point in \mathbb{R} is a cluster point of \mathbb{Q} .

4. In the lecture we have shown

any Cauchy sequence $(x_n)_{n \in \mathbb{N}} \subset \mathbb{R}$ has a limit in \mathbb{R} , i.e. there exists $x \in \mathbb{R}$ with $\lim_{n \rightarrow \infty} x_n = x$. (1)

The same statement is false in \mathbb{Q} , the following is false:

any Cauchy sequence $(x_n)_{n \in \mathbb{N}} \subset \mathbb{Q}$ has a limit in \mathbb{Q} , i.e. there exists $x \in \mathbb{Q}$ with $\lim_{n \rightarrow \infty} x_n = x$. (2)

(a) Give a counterexample to (2).

(b) Which part of the proof of (1) (from the lecture) fails when we attempt to prove (2)?

(Reference for this problem: Theorem 2.4.5 on pg. 74)