The Time Evolution of a Wave Function

• A "system" refers to an electron in a potential energy well, e.g., an electron in a one-dimensional infinite square well. The system is specified by a given Hamiltonian.

• Assume all systems are <u>isolated</u>.

• Assume all systems have a time-independent Hamiltonian operator $\hat{H}$.

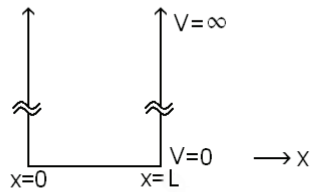
• TISE and TDSE are abbreviations for the time-independent Schrödinger equation and the time-dependent Schrödinger equation, respectively.

• The symbol $\sum$ in all questions denotes a sum over a complete set of states.

PART A

• Information for questions (I)-(VI)

In the following questions, an electron is in a one-dimensional infinite square well of width L. (The stationary states are $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$, and the allowed energies are $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, where $n = 1, 2, 3...$)



(I) Suppose the wave function for an electron at time $t = 0$ is given by $\psi(x, 0) = \sqrt{2/L}\sin(5\pi x/L)$. Which one of the following is the wave function at time t?

(a) $\psi(x, t) = \sqrt{\frac{2}{L}}\ \sin(5\pi x/L)\ \cos(E_5 t/\hbar)$

(b) $\psi(x, t) = \sqrt{\frac{2}{L}}\ \sin(5\pi x/L)\, e^{-iE_5 t/\hbar}$

(c) Both (a) and (b) above are appropriate ways to write the wave function.

(d) None of the above.

(II) The wave function for an electron at time $t = 0$ is given by $\psi(x, 0) = \sqrt{\frac{2}{L}}\ \sin(5\pi x/L)$. Which one of the following is true about the probability density, $|\psi(x, t)|^2$, after time t?

(a) $|\psi(x, t)|^2 = \frac{2}{L}\ \sin^2(5\pi x/L)\ \cos^2(E_5 t/\hbar)$.

(b) $|\psi(x, t)|^2 = \frac{2}{L}\ \sin^2(5\pi x/L)\, e^{-i2E_5 t/\hbar}$.

(c) $|\psi(x, t)|^2 = \frac{2}{L}\ \sin^2(5\pi x/L)$ which is time-independent.

(d) None of the above.

(III) Now suppose that the wave function for an electron at time $t = 0$ is given by $\psi(x, 0) = A \sin^5(\pi x/L)$ where A is a suitable normalization constant. Which one of the following is the wave function at time t?

(a) $\psi(x, t) = A\ \sin^5(\pi x/L)\ \cos(E_5 t/\hbar)$

(b) $\psi(x, t) = A\ \sin^5(\pi x/L)\, e^{-iE_5 t/\hbar}$

(c) Both (a) and (b) above are appropriate ways to write the wave function.

(d) None of the above.

(IV) The wave function for an electron at time $t = 0$ is given by $\psi(x, 0) = A\ \sin^5(\pi x/L)$ where A is a suitable normalization constant. Which one of the following is true about the probability density $|\psi(x, t)|^2$ after time t?

(a) $|\psi(x, t)|^2 = |A|^2\ \sin^{10}(\pi x/L)\ \cos^2(E_5 t/\hbar)$.

(b) $|\psi(x, t)|^2 = |A|^2\ \sin^{10}(\pi x/L)\, e^{-i2E_5 t/\hbar}$.

(c) $|\psi(x, t)|^2 = |A|^2\ \sin^{10}(\pi x/L)$ which is time-independent.

(d) None of the above.

Open the simulation by double-clicking the green arrow associated with this exercise. Shown is the wave function for an electron in the one-dimensional infinite square well at time $t = 0$ given by $\psi(x, 0) = \sqrt{\frac{2}{L}}\ \sin(5\pi x/L)$ for which in the simulation $\hbar = 2m = L = 1$. Click on the time-development to evolve the wave function in time.

(V) Does $|\psi(x, t)|^2$ for a given x change with time in the simulation?

(a) Yes.

(b) No.

(c) I do not know what I should be looking at in the simulation.

Next, choose the simulation that shows the wave function for an electron in the one-dimensional infinite square well at time $t = 0$ given by $\psi(x, 0) = A \sin^5(\pi x/L)$.

Click on the time-development to evolve the wave function in time.

(VI) Does $|\psi(x, t)|^2$ for a given x change with time in the simulation?

(a) Yes.

(b) No.

(c) I do not know what I should be looking at in the simulation.

Try to reconcile your previous responses with what you observed in the simulation. Remember to begin the simulation by double-clicking the green arrow associated with this exercise.

You watched the time-development simulation for two initial wave functions $\psi(x, 0) = \sqrt{\frac{2}{L}} \sin(5\pi x/L)$ and $\psi(x, 0) = A \sin^5(\pi x/L)$ and noted that $|\psi(x, t)|^2$ for the first initial wave function does not depend on time, whereas it does depend on time for the second wave function.

PART B

Let's think systematically about time evolution of a wave function by working through the following questions.

(1) Choose all of the following statements that are correct about the time evolution of a general wave function:

(I) The time evolution of a general wave function is governed by the Hamiltonian operator for that system.

(II) The time evolution of a general wave function is governed by the time-independent Schrödinger equation (TISE): $\hat{H}\psi_n(x) = E_n\psi_n(x)$.

(III) A general wave function $\psi(x, 0)$ at time $t = 0$ will evolve into $\psi(x, t) = e^{-iEt/\hbar}\,\psi(x, 0)$ at time t.

(a) (I) only
(b) (II) only
(c) (III) only
(d) (I) and (II) only
(e) (I) and (III) only

(2) Choose all of the following statements that are correct about the time evolution of a general wave function:

(I) The time evolution of a wave function is governed by the time-dependent Schrödinger equation (TDSE).

(II) Given $\Psi(x, 0)$ and the Hamiltonian, $\hat{H}$, for a system, we can determine the wave function at time t by first calculating the momentum eigenstates.

(III) Given $\Psi(x, 0)$ and the Hamiltonian, $\hat{H}$, we can determine the wave function at time t if we first expand $\Psi(x, 0)$ in terms of the energy eigenstates.

(a) (I) only
(b) (II) only
(c) (III) only
(d) (I) and (II) only
(e) (I) and (III) only

(3) Choose all of the following statements that are important steps in evaluating the wave function at time t given $\Psi(x, 0)$ at time $t = 0$:

(I) Solve the TISE to obtain the stationary states $\psi_n(x)$ and the allowed energies E_n.

(II) Solve the TDSE to obtain the stationary states $\psi_n(x)$ and the allowed energies E_n.

(III) Write $\Psi(x, 0)$ as a linear superposition of the eigenstates of *any* operator, $\hat{A}$, corresponding to a physical observable: $\Psi(x, 0) = \sum c_n \psi_n^{(\hat{A})}(x)$.

(a) (I) only

(b) (II) only

(c) (III) only

(d) (I) and (III) only

(e) (II) and (III) only

(4) Choose all of the following statements that are correct:

(I) The stationary states for a Hamiltonian, $\hat{H}$, form a complete set of states.

(II) Any allowed wave function can be written as a linear superposition of stationary states: $\Psi(x) = \sum c_n \psi_n(x)$

(III) c_n in (II) above can be evaluated using orthogonality of wave functions.

Choose all of the above statements that are correct.

(a) (I) only

(b) (II) only

(c) (III) only

(d) (I) and (II) only

(e) All of the above.

ANSWERS

(1) Correct answer: (a)

Reasoning: (I) is correct because the Hamiltonian operator $\hat{H}$ governs the time evolution of a wave function.

(II) is incorrect because the time evolution of a wave function is governed by the time-dependent Schrödinger equation (TDSE): $i\hbar \frac{\partial \psi(x,t)}{\partial t} = \hat{H}\psi(x,t)$.

(III) is incorrect because only for a stationary state (which is a solution of $\hat{H}\psi_n(x) = E_n\psi_n(x)$) does the wave function evolve in time via a simple phase factor given by $\psi_n(x,t) = e^{-iE_nt/\hbar}\,\psi_n(x)$. E_n is the n^{th} allowed energy for the electron. A general state at time $t = 0$ *may not be a stationary state*.

(2) Correct answer: (e)

Reasoning: (I) is correct because the Hamiltonian for a system governs the time evolution via the TDSE.

(II) is incorrect because in order to find $\Psi(x,t)$, we need to know the energy eigenstates (not the momentum eigenstates) because the time evolution of a wave function is governed by the Hamiltonian $\hat{H}$ and not the momentum operator. Only in special situations, e.g., for a free particle, momentum eigenstates are the same as energy eigenstates.

(III) is correct because once $\Psi(x,0)$ is written as a linear superposition of the energy eigenstates (stationary states), the wave function at time t can easily be determined by evolving each stationary state in time via an appropriate phase factor $e^{-iE_nt/\hbar}\,\psi_n(x)$. Note that each stationary state will have a different phase factor if the energy E_n for those states is different. Mathematically, $\Psi(x,t) = e^{-i\hat{H}t/\hbar}\,\Psi(x,0) = e^{-i\hat{H}t/\hbar}\,[\sum c_n\,\psi_n] = \sum c_n\,e^{-iE_nt/\hbar}\,\psi_n$ where the time-evolution operator $e^{-i\hat{H}t/\hbar}$ acting on each energy eigenstate yields the corresponding phase $e^{-iE_nt/\hbar}$. Note that $H\Psi \neq E\Psi$ although $H\psi_n = E_n\psi_n$.

(3) Correct answer: (a)

Reasoning: (I) is correct because solving the TISE, $\hat{H}\psi_n(x) = E_n\psi_n(x)$, gives the stationary states (states of definite energy) and the allowed energies E_n.
(II) is incorrect because stationary states (energy eigenstates) and the allowed energies are obtained by solving TISE.
(III) is incorrect because the time-development is governed by the $\hat{H}$ operator via TDSE. Therefore, the initial state should be expanded in terms of the stationary states (energy eigenstates) and not eigenstates of *any* operator corresponding to a physical observable.

(4) Correct answer: (e)

Reasoning: (I) is correct because stationary states are the eigenstates of the Hamiltonian (energy eigenstates).
(II) is correct because the wave function can be expanded in terms of a complete set of states.
(III) is correct because c_n is evaluated by multiplying both sides of $\Psi(x) = \sum c_n\psi_n(x)$ by $\psi_m^\star(x)$, integrating over all space, and using orthogonality of stationary states: $\int \psi_m^\star(x)\psi_n(x)dx = \delta_{mn}$. This yields $c_n = \int \psi_n^*(x)\,\Psi(x)\,dx$.

Summary: The following procedure can be used to calculate the wave function $\Psi(x,t)$ at time t given the wave function $\Psi(x,0)$ at time $t=0$ for an electron in a system with a given Hamiltonian $\hat{H}$.

- FIRST: Solve the TISE for that system to find the stationary states $\psi_n(x)$ and allowed energies E_n.
- SECOND: Write the given initial wave function as a linear superposition of the stationary states: $\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x)$.
- THIRD: To find c_n's for each $\psi_n(x)$, multiply both sides of $\Psi(x,0) = \sum c_n \, \psi_n(x)$ by $\psi_m^\star(x)$, integrate over all space, and use orthogonality of stationary states: $\int \psi_m^\star(x)\psi_n(x)\,dx = \delta_{mn}$
- FOURTH: Tack on the time dependence with each stationary state component to obtain the time dependence of the general state: $\Psi(x,t) = \sum_{n=1}^{\infty} c_n \, e^{-iE_n t/\hbar} \, \psi_n(x)$.

Flow chart for computing the time evolution of a state

Given $\Psi(x,0)$ and the Hamiltonian $\hat{H}$ Find $\Psi(x,t)$

$\hat{H}\psi_m = E_m\psi_m$ Solve time-indep. Schröd. eq.

↓ Time dependence of stationary states

Discrete | Continuous

$\psi_m(x,t) = \exp(-iE_m t/\hbar)\,\psi_m(x)$ | $\psi_k(x,t) = \exp(-iE(k)t/\hbar)\,\psi_k(x)$

↓ Expand $\Psi(x,0)$ in terms of stationary states ↓

$\Psi(x,0) = \sum_n c_n\psi_n(x)$ | $\Psi(x,0) = \int C(k)\,\psi_k(x)\,dk$

↓ Find c_m or $c(k)$ using orthogonality ↓

$c_m = \int \psi_m^*(x)\,\Psi(x,0)\,dx$ | $c(k) = \int \psi_k^*(x)\,\Psi(x,0)\,dx$

↓ General solution using superposition ↓

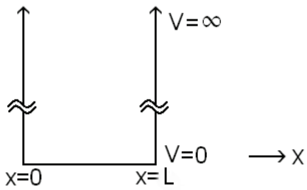
$$\Psi(x,t) = \sum_n c_n \, \psi_n(x,t) = \sum_n c_n \, \exp(-iE_n t/\hbar)\,\psi_n(x)$$

$$\Psi(x,t) = \int c(k)\psi_k(x,t)\,dk = \int c(k)\,\exp(-iE(k)t/\hbar)\,\psi_k(x)\,dk$$

PART C

• **Information for the next four questions**

In the following four questions, an electron is in a one-dimensional infinite square well of width L. (The stationary states are $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$, and the allowed energies are $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, where $n = 1, 2, 3...$)



(1.1) The wave function of the electron at time $t = 0$ is given by $\Psi(x, 0) = \sqrt{\frac{2}{7}}\,\psi_1(x) + \sqrt{\frac{5}{7}}\,\psi_2(x)$ where $\psi_1(x)$ and $\psi_2(x)$ are the ground state and first excited state. Which one of the following is the wave function $\Psi(x, t)$ at time t?

(a) $e^{-iEt/\hbar}\,\Psi(x, 0)$

(b) $e^{-Et/\hbar}\,\Psi(x, 0)$

(c) $e^{-ixt/\hbar}\,\Psi(x, 0)$

(d) $\sqrt{\frac{2}{7}}\,\psi_1(x + \omega t) + \sqrt{\frac{5}{7}}\,\psi_2(x + \omega t)$ where $E = \hbar\omega$

(e) None of the above.

(1.2) Calculate $|\Psi(x, t)|^2$ for the above wave function. Does it depend on time? Explain your reasoning.

Now open the simulation (remember to double-click the green arrow) and choose the initial wave function $\Psi(x, 0) = \sqrt{\frac{2}{7}}\,\psi_1(x) + \sqrt{\frac{5}{7}}\,\psi_2(x)$. Watch the time evolution of $|\Psi(x, t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.

(2) An electron is confined in a one-dimensional infinite square well. Choose one of the following statements that is correct about whether $\psi(x) = A\ \sin^2(\pi x/L)$ is an allowed wave function for the electron. (A is a suitable normalization constant):

(a) It is an allowed wave function.

(b) It is not allowed because it does not satisfy $\hat{H}\psi = E\psi$ where $\hat{H}$ is the Hamiltonian operator.

(c) It is not allowed because it is not a linear function but the Schrödinger equation is linear.

(d) It is not allowed because it is not an energy eigenfunction nor is it a linear superposition of energy eigenfunctions.

(e) $A\ \sin^2(\pi x/L)$ is an allowed wave function for two electrons but not for one electron.

(3) The initial wave function for an electron in a one-dimensional infinite square well at $t = 0$ is given by $\Psi(x, 0) = A \sin^2(\pi x/L)$. Choose all of the following statements related to $\Psi(x, t)$ at time t that are correct.

(I) $\Psi(x,t) = A\, e^{-iE_n t/\hbar} \sin^2(\pi x/L)$

(II) $\Psi(x,t) = \sum c_n\, e^{-iE_n t/\hbar}\, \psi_n(x)$ where the expansion coefficients $c_n = \int \psi_n^*(x)\, A \sin^2(\pi x/L)\, dx$.

(III) $\Psi(x,t) = \sum c_n \sin^2(\pi x/L)\, e^{-iE_n t/\hbar}\, \psi_n(x)$ where the expansion coefficients $c_n = \int \psi_n^*(x)\, A \sin^2(\pi x/L)\, dx$.

(a) (I) only

(b) (II) only

(c) (III) only

(d) (I) and (II) only

(e) (I) and (III) only

(4) An electron is confined in a one-dimensional infinite square well. The wave function at time $t = 0$ is denoted by $\Psi(x, 0)$. $|\Psi(x_0, t)|^2\, dx$ is the probability of finding the electron in the narrow range between x_0 and $x_0 + dx$ at time t. Choose all of the following statements that are correct:

(I) If $\Psi(x, 0)$ is a stationary state, $|\Psi(x_0, t)|^2\, dx$ will not depend on time.

(II) If $\Psi(x, 0)$ is localized in space around a position x_0, $|\Psi(x_0, t)|^2\, dx$ will not depend on time.

(III) If $\Psi(x, 0)$ is in an energy eigenstate, $|\Psi(x_0, t)|^2\, dx$ will not depend on time.

(a) (I) only

(b) (II) only

(c) (III) only

(d) (I) and (III) only

(e) All of the above.

ANSWERS

(1.1) Correct answer: (e)

Reasoning: Using the above flow chart, we should first expand $\Psi(x,0)$ in terms of the stationary states: $\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x)$ and determine the coefficients c_n. Fortunately, this $\Psi(x,0)$ is already given in the form $\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x) = \sqrt{\frac{2}{7}}\,\psi_1(x) + \sqrt{\frac{5}{7}}\,\psi_2(x)$ and simply by observation, the coefficients $c_1 = \sqrt{\frac{2}{7}}$, $c_2 = \sqrt{\frac{5}{7}}$, and all the other c_n are zero. Since each stationary state $\psi_n(x)$ evolves independently via a phase factor $e^{-iE_nt/\hbar}$, $\Psi(x,t) = \sqrt{\frac{2}{7}}\,e^{-iE_1t/\hbar}\,\psi_1(x) + \sqrt{\frac{5}{7}}\,e^{-iE_2t/\hbar}\,\psi_2(x)$.

Note that the time-dependent phase factors cannot be factored out because they are different for different stationary states.

(2) Correct answer: (a)

Reasoning: (b) is incorrect because the allowed wave functions need not satisfy the TISE $\hat{H}\psi = E\psi$. Rather, any linear superposition of the stationary states (which are solutions of $\hat{H}\psi = E\psi$) is also an allowed state.

(c) $A\ \sin^2(\pi x/L)$ is a smooth function (no discontinuity or cusp) and satisfies the boundary condition for this system (i.e., $\psi(x=0) = 0$ and $\psi(x=L) = 0$ as can be checked by plugging $x = 0$ and $x = L$ in $\psi(x) = A\ \sin^2(\pi x/L)$). Therefore, $A\ \sin^2(\pi x/L)$ can be written as a linear superposition of stationary states.

(d) $A\ \sin^2(\pi x/L)$ can be written as a linear superposition of stationary states (energy eigenstates).

(e) $A\ \sin^2(\pi x/L)$ is an allowed wave function for a single electron. A two-electron wave function must have different coordinates for the two electrons, e.g., $\Psi(x_1, x_2) = A\sin(\pi x_1/L)\sin(\pi x_2/L)$.

Bottom line: If the wave function is single-valued, normalizable, the wave function and its derivative are continuous everywhere, and it satisfies the boundary condition for a particular system, it is an allowed wave function.

(3) Correct answer: (b)

Reasoning: (I) is incorrect because $\Psi(x,0) = A\,\sin^2(\pi x/L)$ is not a stationary state with energy E_n.
(II) is correct because in evaluating $\Psi(x,t)$, $\Psi(x,0) = A\,\sin^2(\pi x/L)$ should only determine the coefficients $c_n = \int \psi_n^*(x)\,A\,\sin^2(\pi x/L)\,dx$.
(III) is incorrect because $\Psi(x,t) = \sum c_n\,e^{-iE_nt/\hbar}\,\psi_n(x)$.

(4) Correct answer: (d) Stationary state and energy eigenstate have the same meaning.

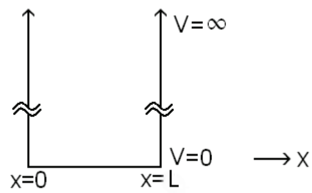
Reasoning: (I) is correct because the stationary states evolve via a simple phase factor: $\Psi(x,t) = e^{-iE_nt/\hbar}\,\psi_n(x)$ and $|\Psi(x_0,t)|^2\,dx = e^{-iE_nt/\hbar}\,\psi_n(x)\,e^{iE_nt/\hbar}\,\psi_n^\star(x) = \psi_n(x)\psi_n^\star(x)$ will not depend on time because the time-dependent phase factor will drop out due to complex conjugation.
(II) is incorrect because a wave function which is very localized in space around a position x_0 cannot be a stationary state. Therefore, we will have to expand the wave function in terms of the stationary states to find the wave function at time t and different stationary states will evolve via different phase factors: $\Psi(x,t) = \sum_n c_n\,\exp(-iE_nt/\hbar)\,\psi_n(x)$. Taking $|\Psi(x,t)|^2\,dx$ will not get rid of the time-dependent factors (as the cross terms do not necessarily vanish) and $|\Psi(x_0,t)|^2\,dx$ will change with time.
(III) is correct because a state with definite energy is a stationary state (solution of TISE). Because the stationary states evolve via a simple phase factor: $\Psi(x,t) = e^{-iE_nt/\hbar}\,\psi_n(x)$, $|\Psi(x_0,t)|^2\,dx$ will not depend on time (the time-dependent phase factor will drop out due to complex conjugation).

Practice Worksheet for the Time Evolution of a Wave Function

Given the wave function at time $t = 0$, use the above flow chart to obtain the wave function at time t, $\Psi(x,t)$, for the following cases. In all problems, A is a suitable normalization constant. All final answers can be written in terms of the normalization constant A, the stationary state wave functions $\psi_n(x)$, and the allowed energies E_n.

You NEED NOT evaluate A, $\psi_n(x)$ and E_n. You may also use the flow chart given earlier as needed.

(I) In the following questions, an electron is in a one-dimensional infinite square well of width L.



1. The initial wave function at $t = 0$ is given by $\Psi(x,0) = \sqrt{\frac{1}{8}}\psi_1(x) + \sqrt{\frac{7}{8}}\psi_2(x)$ where $\psi_1(x)$ and $\psi_2(x)$ are the ground state and the first excited state wave functions. Find $\Psi(x,t)$.

2. Calculate $|\Psi(x,t)|^2$ for the above wave function. Does it depend on time? Explain why it should or should not depend on time.

3. Now open the simulation (double-click the green arrow) and choose the initial wave function $\Psi(x,0) = \sqrt{\frac{1}{8}}\psi_1(x) + \sqrt{\frac{7}{8}}\psi_2(x)$. Watch the time evolution of $|\Psi(x,t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.

4. Now consider an initial wave function at $t = 0$ given by $\Psi(x, 0) = A\,x\,(L - x)$. Find $\Psi(x, t)$. You need not evaluate the integrals involved but you must set them up (write all the unknown quantities in terms of A, L, $\psi_n(x)$, and E_n). You need NOT evaluate $\psi_n(x)$ and E_n.

5. Calculate $|\Psi(x, t)|^2$ for the above wave function. Does it depend on time? Explain why it should or should not depend on time.

6. Now open the simulation (double-click the green arrow) which shows the initial wave function $\Psi(x, 0) = Ax(L - x)$. Watch the time evolution of $|\Psi(x, t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.

7. The initial wave function at $t = 0$ is given by $\Psi(x, 0) = A \sin^3(\pi x/L)$. Find $\Psi(x, t)$. You need not evaluate the integrals involved but you must set them up (write all the unknown quantities in terms of A, L, $\psi_n(x)$, and E_n). You need NOT evaluate $\psi_n(x)$ and E_n.

8. Calculate $|\Psi(x, t)|^2$ for the above wave function. Does it depend on time? Explain why it should or should not depend on time.

9. Now open the simulation (double-click the green arrow) which shows the initial wave function $\Psi(x, 0) = A \sin^3(\pi x/L)$. Watch the time evolution of $|\Psi(x, t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.

(II) In the following problems, an electron is in a one-dimensional harmonic oscillator well with potential energy $V(x) = m\omega^2 x^2/2$ and with $E_n = (n + \frac{1}{2})\hbar\omega$ for $n = 0, 1, 2, \ldots$.

1. The electron is in the ground state at $t = 0$. The ground state wave function is given by $\Psi(x, 0) = Ae^{-\alpha x^2}$ where $\alpha = \frac{m\omega}{2\hbar}$. Find $\Psi(x, t)$.

2. Calculate $|\Psi(x, t)|^2$ for the above wave function. Does it depend on time? Explain why it should or should not depend on time.

3. Now open the simulation which shows the initial wave function to be $\Psi(x, 0) = Ae^{-\alpha x^2}$. Watch the time evolution of $|\Psi(x, t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.

4. The initial wave function at time $t = 0$ is given by $\Psi(x, 0) = \sqrt{\frac{1}{3}}\,\psi_1(x) + e^{i\theta}\sqrt{\frac{2}{3}}\,\psi_2(x)$ where $\psi_1(x)$ and $\psi_2(x)$ are the ground state and first excited state wave functions for the one-dimensional harmonic oscillator well. θ is a real constant. Find $\Psi(x, t)$.

5. Calculate $|\Psi(x, t)|^2$ for the above wave function. Does it depend on time? Explain why it should or should not depend on time.

6. Now open a simulation by double-click, selecting an angle, θ, which shows the initial wave function to be $\Psi(x, 0) = \sqrt{\frac{1}{3}}\,\psi_1(x) + e^{i\theta}\sqrt{\frac{2}{3}}\,\psi_2(x)$. Watch the time evolution of $|\Psi(x, t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.

(III) Consider an electron which is free to move in a one-dimensional space (free particle). Solutions to the TISE for this problem are of the form e^{ikx}, where $k = \sqrt{2mE/\hbar^2}$ is a continuous variable (and so is E).

1. The initial wave function at $t = 0$ is given by $\Psi(x, 0) = Ae^{-\alpha x^2}$ (α is a positive real constant with the dimension of inverse square length). Find $\Psi(x, t)$. You need not evaluate the integrals involved but you must set them up (write all the unknown quantities in terms of A, α, $\psi_n(x)$, and E_n). You need NOT evaluate $\psi_n(x)$ and E_n.

2. Calculate $|\Psi(x, t)|^2$ for the above wave function. Does it depend on time? Explain why it should or should not depend on time.

3. Now open the simulation which shows the initial wave function of a free particle: $\Psi(x, 0) = Ae^{-\alpha x^2}$. Watch the time evolution of $|\Psi(x, t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.