Ayres Freitas http://www.pitt.edu/~afreitas/phy1370.html

Homework for week of 10/14/2008

Deadline 10/20/2008

Notice:

The midterm exam will be on Wed, Oct 22, 2008 during regular class at 11am.

Problem 1: *(2 points)*

Given the two matrices

$$\mathsf{A} = \begin{pmatrix} 1 & 0 & i \\ 0 & 0 & -i \\ -i & i & -1 \end{pmatrix}, \qquad \mathsf{B} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix},$$

calculate $[\mathsf{A}, \mathsf{B}]$, $\mathsf{A}^\dagger$, $\mathsf{B}^\dagger$. Which of the matrices are symmetric, hermitian, and/or unitary?

Problem 2: *(3 points)*

a) For what range of $r \in \mathbb{R}$ is the function $f(x) = x^r$ in Hilbert space, on the interval (0,1)?

b) For the specific case $r = 1/2$, is $f(x)$ in Hilbert space? What about $\frac{d}{dx} f(x)$?

Problem 3: *(2 points)*

Show that if $\langle h|\hat{Q}h\rangle = \langle \hat{Q}h|h\rangle$ for all functions h in Hilbert space, then $\langle f|\hat{Q}g\rangle = \langle \hat{Q}f|g\rangle$ for all f, g in the same space.
Hint: first let $h = f + g$, and then $h = f + ig$.

Problem 4: *(4 points)*

The *hermitian conjugate* (or *adjoint*) of an operator $\hat{Q}$ is the operator $\hat{Q}^\dagger$ with

$$\langle f|\hat{Q}g\rangle = \langle \hat{Q}^\dagger f|g\rangle, \qquad \text{for all } f \text{ and } g \text{ in Hilbert space.}$$

a) Find the hermitian conjugates of x, i, and $\frac{d}{dx}$.

b) Construct the hermitian conjugate of the harmonic oscillator raising operator $\hat{a}_+$.

c) Show that $(\hat{Q}\hat{R})^\dagger = \hat{R}^\dagger \hat{Q}^\dagger$.

d) Show that the position operator, $\hat{x} = x$, and the Hamiltonian operator,

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x),$$

are hermitian, i.e. $\hat{x}^\dagger = \hat{x}$ and $\hat{H}^\dagger = \hat{H}$.

Problem 5: *(1 point)*

Show that the eigenfunctions of the hermitian operator $\hat{Q} = id/d\phi$ (see example in the lecture) are orthogonal (for distinct eigenvalues).

Problem 6: *(2 points)*

Consider the operator $\hat{Q} = d^2/d\phi^2$, where (as in the example in the lecture) ϕ is the polar coordinate in two dimensions, i.e. it is restricted to the interval $0 \leq \phi \leq 2\pi$. Is $\hat{Q}$ hermitian? Find its eigenfunctions and eigenvalues. What is the spectrum of $\hat{Q}$? Is the spectrum degenerate?