Ayres Freitas http://www.pitt.edu/~afreitas/phy1370.html

Homework for week of 10/06/2008

Deadline 10/14/2008

Notice:

The midterm exam will be on Wed, Oct 22, 2008 during regular class at 11am.

Monday, Oct 13 is a university holiday. No classes will be held on that day. The quantum mechanics class will meet on Tuesday, Oct 14, at 11am in Allen 106. Classes that are regularly scheduled on Tuesdays will not meet this week.

Problem 1: *(5 points)*

a) Determine the transmission coefficient for a rectangular *barrier*

$$V(x) = \begin{cases} V_0, & \text{for } -a < x < a, \\ 0, & \text{for } |x| > a, \end{cases}$$

with $V_0 > 0$, for the case $E < V_0$.

Answer:

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right).$$

b) What is the probability that an electron with energy 1 eV $= 1.602 \times 10^{-19}$ J and mass 9.109×10^{-31} kg can tunnel through the isolation barrier of a FET transistor with potential height 3.15 eV and thickness 1 nm?

What is the probability that a billiard ball with mass 0.2 kg and speed 0.5 m/s can tunnel through the wall of the billiard table with height 5 cm and thickness 10 cm? Give the result in the form 10^x, i.e. calculate $\log_{10} T$ and approximate the formula for large arguments of the sinh.

Problem 2: *(5 points)*

Consider the following sets of functions and determine whether they constitute a vector space. If so, suggest a convenient basis and give the dimension of the space. If not, which defining properties are not satisfied?

a) All polynomials in x (with complex coefficients) of degree less than N.

b) Only even polynomials of degree less than N.

c) All polynomials of degree less than N with the leading coefficient (i.e. the coefficient of x^{N-1}) equal 1.

d) All polynomials of degree less than N that have the value 0 at $x = 1$.

e) All polynomials of degree exactly N.

Problem 3*: *(5 bonus points)*

Only for the most simple potentials, like the examples in the lecture and the book, the Schrödinger equation can be solved analytically. It can, however, be solved numerically for any finite potential $V(x)$.

Solve the Schrödinger equation for the scattering of a wave with fixed momentum $\psi(x) = e^{ikx}$, incoming from the left, on the potential

$$V(x) = \begin{cases} V_0 \cos \frac{\pi x}{2a}, & \text{for } -a < x < a, \\ 0, & \text{for } |x| > a, \end{cases}$$

by implementing the following simple procedure on a computer:

- First observe that we know that

$$\begin{aligned} \psi(x) &= Ae^{ikx} + Be^{-ikx} \quad && \text{for } x < -a, \\ \psi(x) &= Fe^{ikx} && \text{for } x > +a, \end{aligned}$$

  Since the wave function has the simplest form for $x > +a$, it is easiest to evolve it *backwards* from positive to negative values of x.

- Use the values $m = \hbar \times \frac{1}{2}$ ns/nm^2, $a = 1$ nm, $V_0 = \hbar \times 2$ ns^{-1}, $E = \frac{1}{2} V_0$, and $k = \sqrt{2mE}/\hbar$. Let all factors of $\hbar$ and time units cancel and work with x is units of nm. Use your favorite programming language (for example Mathematica or Mathlab).

- Start at $x = +10$ nm. Set the initial values (boundary conditions)

$$\psi(x) = Fe^{ikx}, \qquad \psi'(x) = \frac{d}{dx} Fe^{ikx}.$$

  For simplicity, use $F = 1$.

- Now determine $\psi''(x)$ from the Schrödinger equation. Then use the following approximations for obtaining the value of ψ at $x - \epsilon$ for a small value of ϵ:

$$\psi'(x - \epsilon) \approx \psi'(x) - \epsilon\, \psi''(x), \qquad \psi(x - \epsilon) \approx \psi(x) - \epsilon\, \psi'(x).$$

  Now set x to $x - \epsilon$ and keep repeating this step until you reach $x = -10$ nm. It is important to make sure that ϵ is small enough, otherwise the algorithm can create artificial singularities.

Hand in a copy of your code and plots of Re$\{\psi(x)\}$ and Im$\{\psi(x)\}$.

Note: This is a very crude algorithm. Much more sophisticated algorithms are known for solving differential equations numerically. However, this simple method should give you a good intuitive understanding of what is happening.