Ayres Freitas http://www.pitt.edu/~afreitas/phy1370.html

Homework for week of 09/22/2008

Deadline 09/29/2008

Problem 1: *(2 points)*

a) Construct the stationary state $\psi_2(x)$ of the harmonic oscillator using the algebraic method.

b) Check the orthogonality of ψ_0, ψ_1, and ψ_2 by explicit integration.
Hint: If you exploit the even-ness and odd-ness of the functions, there is really only one integral left to do.

Problem 2: *(4 points)*

a) Compute $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$ for the state ψ_0 of the harmonic oscillator.
Hint: here and in other problems involving the harmonic oscillator it simplifies matters to introduce the variable $\xi \equiv \sqrt{m\omega/\hbar}\, x$ and the constant $\alpha \equiv (m\omega/(\pi\hbar))^{1/4}$.

b) Check the uncertainty principle for this state.

c) Compute $\langle T \rangle$ (the average kinetic energy) and $\langle V \rangle$ (the average potential energy) for this state, using the results from (a). Is their sum what you would expect?

Problem 3: *(3 points)*
Find $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, and $\langle T \rangle$, for the nth stationary state of the harmonic oscillator, using the method of example 2.5 in the book. Check that the uncertainty principle is satisfied.

Problem 4: Relations for Hermite polynomials *(4 points)*

a) The *Rodrigues formula* says that

$$H_n(\xi) = (-1)^n e^{\xi^2} \left(\frac{d}{d\xi}\right)^n e^{-\xi^2}. \tag{1}$$

Use it to derive H_3 and H_4.

b) Use the recursion relation

$$H_{n+1}(\xi) = 2\xi H_n(\xi) - 2nH_{n-1}(\xi), \tag{2}$$

together with the result from (a), to obtain H_5 and H_6.

c) Prove that

$$H_n'(\xi) = 2nH_{n-1}(\xi), \tag{3}$$

using $\hat{a}_-\psi_n = \sqrt{n}\,\psi_{n-1}$ and equation [2.85] in the book.

"Actually I started out in quantum mechanics, but somewhere along the way I took a wrong turn."