PHYS 1370 **Introduction to Quantum Mechanics I** **Fall 2008**

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Homework for week of 09/15/2008

Deadline 09/22/2008

Problem 1: *(2 points)*
Show that E must exceed the minimum value of $V(x)$, for every normalizable solution to the time-independent Schrödinger equation (TISE). What is the classical analog to this statement?
Hint: Write the TISE in the form

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar}[V(x) - E]\psi \; ; \tag{1}$$

if $E < V_{\rm min}$, then ψ and its 2nd derivative always have the same sign—argue that such a function cannot be normalized.

Problem 2: *(3 points)*
Calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, σ_x and σ_p, for the nth stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit?

Problem 3: *(5 points)*
A particle in the infinite square well has as its initial state an even mixture of the first two stationary states:

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)]. \tag{2}$$

a) Normalize $\Psi(x, 0)$, *i.e.* find A. It helps to exploit the orthonormality of ψ_1 and ψ_2.

b) Find $\Psi(x, t)$ and $|\Psi(x, t)|^2$. Express the latter as a sinusoidal function of time, as in Example 2.1 in the book. To simplify the result, let $\omega \equiv \pi^2\hbar/(2ma^2)$.

c) Compute $\langle x \rangle$. Notice that it oscillates in time. What is the angular frequency and amplitude of the oscillation?

d) Compute $\langle p \rangle$. Use the method that seems easiest to you.

e) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation value of $\hat{H}$. How does it compare with E_1 and E_2?

Problem 4: *(4 points)*

A particle in the infinite square well has the initial wave function

$$\Psi(x,0) = \begin{cases} Ax, & 0 \le x \le a/2, \\ A(a-x), & a/2 \le x \le a. \end{cases} \tag{3}$$

a) Sketch $\Psi(x,0)$, and determine A.

b) Find $\Psi(x,t)$.

c) What is the probability that a measurement of the energy would yield the value E_1?

d) Find the expectation value of the energy.

Problem 5: *(3 points)*

A particle of mass m is in the ground state of the infinite square well. Suddenly the well expands to twice its original size—the right wall shifting from a to $2a$—leaving the wave function (momentarily) undisturbed. The energy of the particle is now measured.

a) What is the most probable result? What is the probability of getting that result?

b) What is the *next* most probable result, and what is its probability?

c) What is the expectation value of the energy?
Hint: If you find yourself confronted with an infinite series, try another method.