PHYS 1370 **Introduction to Quantum Mechanics I** **Fall 2008**

Ayres Freitas http://www.pitt.edu/~afreitas/phy1370.html

Homework for week of 09/03/2008

Deadline 09/08/2008

Problem 1: *(3 points)*

Consider the olympic athletic team of Utopistan. The team has twenty members; one won six medals, one won five medals, three won four medals, two won three medals, three won two medals, five won one medals, and the rest did not get any medal.

a) Compute the average number of medals each athlete won, *i.e.* compute $\langle n \rangle$ with n being the number of medals.

b) Calculate $\langle n^2 \rangle$, $\langle n \rangle^2$, and $\sigma = \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$.

c) Find the standard deviation via the "pedestrian" method, *i.e.* determine Δn for each n and use $\sigma^2 = \langle (\Delta n)^2 \rangle$.

d) Compare your results in (b) and (c).

Problem 2: *(2 points)*

For the *falling rock* example in the lecture, with

$$x(t) = \tfrac{1}{2} g t^2, \qquad \rho(x) = \frac{1}{2\sqrt{hx}}, \quad (0 \le x \le h), \tag{1}$$

a) Find the standard deviation of the distribution.

b) What is the probability that a photograph, selected at random, would show a distance x more than one standard deviation away from the average?

Problem 3: *(3 points)*

Consider the *gaussian* distribution

$$\rho(x) = A\, e^{-\lambda (x-a)^2}, \tag{2}$$

where A, a, and λ are positive real constants. Look up any integrals you need.

a) Determine A so that $\rho(x)$ is properly normalized.

b) Find $\langle x \rangle$, $\langle x^2 \rangle$, and σ.

c) Sketch the graph of $\rho(x)$.

Problem 4: *(3 points)*

The needle of a broken car speedometer is free to swing, and bounces perfectly off the pins at either end, so that if you give it a flick it is equally likely to come to rest at any angle θ between 0 and π.

a) What is the probability density $\rho(\theta)$?
Hint: $\rho(\theta)\, d\theta$ is the probability that the needle will comes to rest between θ and $\theta + d\theta$. Graph $\rho(x)$ as a function of θ, from $-\pi/2$ to $3\pi/2$. Make sure that the total probability is 1.

b) Compute $\langle\theta\rangle$, $\langle\theta^2\rangle$, and σ for this distribution.

c) Compute $\langle\sin\theta\rangle$, $\langle\cos\theta\rangle$, and $\langle\cos^2\theta\rangle$.

Problem 5: *(2 points)*

We consider the same device as in the previous problem, but this time we are interested in the x-coordinate of the needle point—that is, the "shadow," or "projection," of the needle on the horizontal line.

a) What is the probability density $\rho(x)$? Graph $\rho(x)$ as a function of x, from $-2r$ to $2r$, where r is the length of the needle. Make sure that the total probability is 1.
Hint: You know from Problem 4 the probability that θ is in a interval $d\theta$; the question is, what interval dx corresponds to the interval $d\theta$?

b) Compute $\langle x\rangle$, $\langle x^2\rangle$, and σ, for this distribution. Explain how you could have obtained these results from part (c) of Problem 4.