Ayres Freitas http://www.pitt.edu/~afreitas/phy1370.html

Homework for week of 11/17/2008

Deadline 11/24/2008

Problem 1: *(3 points)*
An electron is in the spin state

$$|\chi\rangle = A\big[(1+i)\,|\uparrow\rangle_z + (1-i)\,|\downarrow\rangle_z\big]$$

a) Determine the normalization constant A. What is the spinor in S_z-basis?

b) Find the expectation values and standard deviations of S_x, S_y, and S_z.

c) Check that your results satisfy all three uncertainty principles (eq. [4.100] in the book and its cyclic permutations).

Problem 2: *(3 points)*
Find the eigenvalues and eigenspinors of S_y. If you measured S_y on a particle in the general state $|\chi\rangle = a\,|\uparrow\rangle_z + b\,|\downarrow\rangle_z$, what values can you get, and with what probabilities? Check that the probabilites add up to 1.

Problem 3: *(2 points)*
Construct the spin matrices for a particle of spin 1. Start with the eigenstates of $\hat{S}_z$ and follow the procedure in the lecture/book for spin 1/2.

Problem 4: *(2 points)*
Let us use the abbreviation SGZ± for a Stern-Gerlach experiment with magnetic field gradient in the positive/negative z direction.

a) For an atom with total spin quantum number $S = 1$, into how many spatially separate parts will an arbitrary wave function split upon passing through a SGZ ?

b) How many spatially separate parts would you expect instead if $S = 0$?

c) Suppose that you prepare silver atoms (with $S = \frac{1}{2}$) with an initial state $|\psi\rangle = \frac{3}{5}|\uparrow\rangle_z + \frac{4}{5}|\downarrow\rangle_z$ and let them pass through SGZ−. If you place a single detector in the path of the "spin-up" component what is the probability for a detection? What spin state can we get from the down channel of the SGZ− ?

Problem 5: *(3 points)*

Study the following conversations. In each case, with whom do you agree? If you do not agree with someone, explain why that person is not correct.

a) **Donald:** There is no difference between silver atoms in a "pure" state given by $\frac{1}{\sqrt{2}}\big(|\uparrow\rangle_z + |\downarrow\rangle_z\big)$ and an unpolarized mixture in which half of the atoms are in the $|\uparrow\rangle_z$ state and half are in $|\downarrow\rangle_z$ state. If I had sent atoms in the superposition state $\frac{1}{\sqrt{2}}\big(|\uparrow\rangle_z + |\downarrow\rangle_z\big)$ through the SGZ of problem 3c), half of them in the $|\uparrow\rangle_z$ state would have registered in the "up" detector and half of them in the $|\downarrow\rangle_z$ state would have been collected. There is no way to distinguish a mixture from a superposition.

   Daisy: I disagree. We can distinguish between the superposition and mixture you mentioned by passing each of them through an SGX.

b) **Mad Hatter:** The eigenstates of $\hat{S}_x$ are orthogonal to the eigenstates of $\hat{S}_z$.

   Alice: I disagree. The different eigenstates of $\hat{S}_x$ are orthogonal to one another, however. In fact, the orthogonal eigenstates of $\hat{S}_x$ form one basis for the vector space for the spin degrees of freedom and the orthogonal eigenstates of $\hat{S}_z$ form another basis.

c) **Sokrates:** If an atom in the state $|\uparrow\rangle_z$ were sent through an SGX$-$, the atom will be unaffected by the field gradient. The atom cannot feel a force because its state $|\uparrow\rangle_z$ is perpendicular to the magnetic field.

   Plato: I disagree. You are getting confused between a state in the Hilbert space and the magnetic field in the physical space. We can write $|\uparrow\rangle_z$ in terms of the eigenstates of $\hat{S}_x$ and the magnetic field gradient in the x direction will spatially separate the $|\uparrow\rangle_x$ and $|\downarrow\rangle_x$ components.