PHYS 1370 **Introduction to Quantum Mechanics I** **Fall 2008**

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Homework for week of 11/10/2008

Deadline 11/17/2008

Problem 1: *(4 points)*

Find the stationary states and energy eigenvalues of the two-dimensional infinite square well,

$$V(x,y) = \begin{cases} 0 & \text{for } |x| \leq a \text{ and } |y| \leq a, \\ \infty & \text{otherwise,} \end{cases}$$

by using separation of variables in x, y.
Determine the degeneracies of the first six energy levels.

Problem 2: *(3 points)*

A particle with mass m is placed in the three-dimensional *finite* spherical well:

$$V(r) = \begin{cases} -V_0 & \text{for } r \leq a, \\ 0 & \text{for } r > a. \end{cases}$$

Find the ground state, by solving the radial equation with $l = 0$. Show that there is no bound state if $V_0 < \frac{\pi^2\hbar^2}{8ma^2}$.

Problem 3: *(3 points)*

a) Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius.

b) Find $\langle y \rangle$ and $\langle y^2 \rangle$ for an electron in the ground state of hydrogen.
Hint: You can exploit the result of (a) and the symmetry of the ground state.

c) Find $\langle y^2 \rangle$ in the state $n, l, m = 2, 1, 0$.
Note: This state is not spherically symmetric. Translate y to spherical coordinates.

Problem 4: *(4 points)*

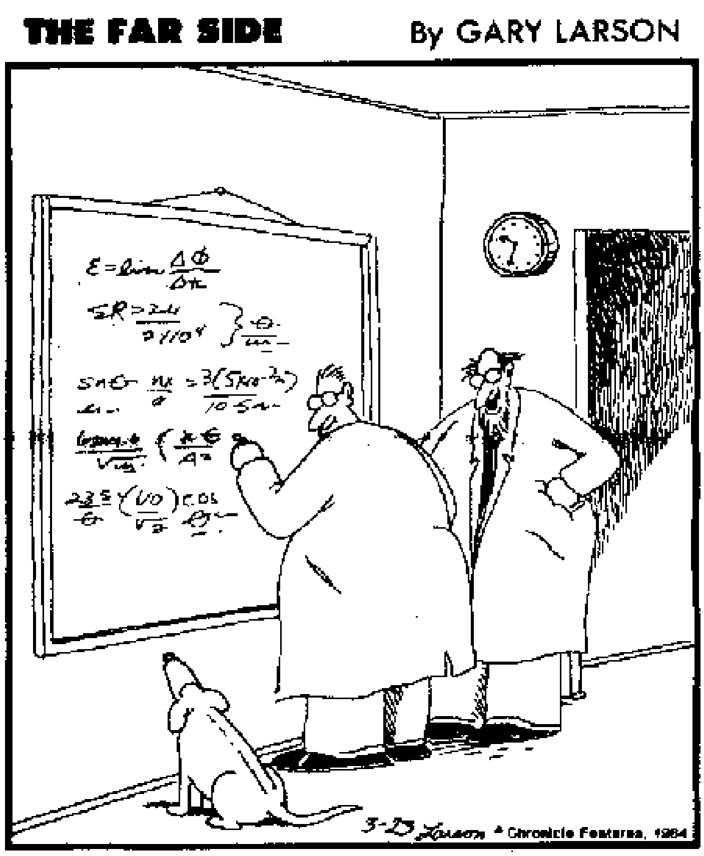
Consider the three-dimensional harmonic oscillator with

$$V(r) = \frac{1}{2} m\omega^2 r^2.$$

a) Use separation of variables in cartesian coordinates to turn this into three one-dimensional oscillators, and determine the allowed energies E_n.

b) Determine the degeneracy $d(n)$ of E_n.

c) Because the three-dimensional harmonic oscillator is spherically symmetric, the Schrödinger equation can also be solved by separation of variables in spherical coordinates. Determine the ground state by solving the radial equation with $l = 0$. Compare your answers in (a) and (c).



"Ohhhhhhh . . . Look at that, Schuster . . . Dogs are so cute when they try to comprehend quantum mechanics."