Ayres Freitas http://www.pitt.edu/~afreitas/phy1370.html

Homework for week of 11/03/2008

Deadline 11/10/2008

Problem 1: *(3 points)*

The Hamiltonian of some two-level system is

$$\hat{H} = h\left(|1\rangle\langle 1| - |2\rangle\langle 2| + i|1\rangle\langle 2| - i|2\rangle\langle 1|\right),$$

where $|1\rangle, |2\rangle$ is an orthonormal basis and h is a number with dimension of energy. Find its eigenvalues and eigenvectors (as linear combinations of $|1\rangle$ and $|2\rangle$). Give the matrix H representing $\hat{H}$ in the basis of $|1\rangle, |2\rangle$; as well as in the basis of the eigenvectors.

Problem 2: *(5 points)*

a) Calculate the following commutators:

$$[\hat{L}_z, \hat{x}] \qquad [\hat{L}_z, \hat{y}] \qquad [\hat{L}_z, \hat{z}] \qquad [\hat{L}_z, \hat{p}_x] \qquad [\hat{L}_z, \hat{p}_y] \qquad [\hat{L}_z, \hat{p}_z]$$

b) Evaluate the commutators $[\hat{L}_z, r^2]$ and $[\hat{L}_z, \hat{p}^2]$, where $r^2 = x^2 + y^2 + z^2$ and $\hat{p}^2 = \hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2$.

c) Show that the Hamilton operator $\hat{H} = \hat{p}^2/(2m) + V(r)$ commutes with all three components of $\hat{\mathbf{L}}$.

d) Show that $d\langle\hat{\mathbf{L}}\rangle/dt = 0$ for any spherically symmetric potential.
Note: This is one form of the quantum statement of conservation of angular momentum.

Problem 3: *(5 points)*

a) What is $\hat{L}_+ Y_l^l$?

b) Use the result of (a), together with

$$\hat{L}_+ = \hbar e^{i\phi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$$

and the fact that $\hat{L}_z Y_l^l = \hbar l Y_l^l$, to determine $Y_l^l(\theta, \phi)$ up to a normalization constant.

c) Check explicitly that your solution for Y_l^l satisfies the angular equation ([4.18] in the book).

d) Check explicitly that the Y_l^l are orthogonal for different values of l.

Problem 4: *(4 points)*

Given the angular function

$$\Upsilon(\theta, \phi) = \frac{1}{2} Y_{1,-1}(\theta, \phi) + \frac{1}{\sqrt{2}} Y_{1,0}(\theta, \phi) + \frac{1}{2} Y_{1,1}(\theta, \phi).$$

Is $\Upsilon(\theta, \phi)$ an eigenfunction of $\hat{L}^2$, $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$?
Determine the expectation values of these operators.