

Homework for week of 08/25/2008

Deadline 09/03/2008

Problem 1:

(1 point)

Show that the Euler-Lagrange equations in the Lagrange formalism

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}_i}, \quad (1)$$

and the canonical equations in the Hamilton formalism

$$\frac{\partial \mathcal{H}}{\partial p_i} = \dot{x}_i, \quad \frac{\partial \mathcal{H}}{\partial x_i} = -\dot{p}_i, \quad \mathcal{H} = \sum_i \dot{x}_i p_i - \mathcal{L} \quad (2)$$

are equivalent.

Note: $\mathcal{H} = \mathcal{H}(x_1, x_2, \dots, p_1, p_2, \dots)$ and $\mathcal{L} = \mathcal{L}(x_1, x_2, \dots, \dot{x}_1, \dot{x}_2, \dots)$ are functions of different variables, so be careful with partial derivatives and use the chain rule where necessary.

Problem 2: Review of basic vector algebra

(3 points)

Consider an elastic collision between billiard ball A with momentum p_A and billiard ball B with momentum p_B . After the collision the momenta are p'_A and p'_B , respectively. The x-, y-, and z-components of the momenta are given by

$$p_A = \begin{pmatrix} -1 - \sqrt{2} \\ -1 + \sqrt{2} \\ 2 \end{pmatrix}, \quad p_B = \begin{pmatrix} \sqrt{2} \\ -\sqrt{2} \\ 0 \end{pmatrix}, \quad p'_A = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \quad (3)$$

- Calculate the angle between the incoming and outgoing direction of B.
- Compute $M \cdot p_B$ where

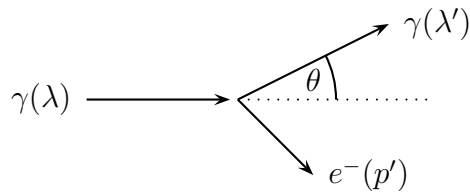
$$M = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 3 & 3 & 1 \end{pmatrix}. \quad (4)$$

What special property does p_B have for M ?

- Determine all eigenvalues of M .

Problem 3: Compton scattering

(2 points)



An incident X-ray photon with wavelength λ hits an electron at rest. The two particles scatter elastically, with the outgoing photon having wavelength λ' and the outgoing electron having momentum p' .

Show that

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos \theta), \quad (5)$$

where θ is the angle between incoming and outgoing photon.

Hint: use energy-momentum conservation and the energy-momentum relation from special relativity:

	photon energy	photon momentum	electron energy	electron momentum
before scattering	$\frac{hc}{\lambda}$	$\frac{h}{\lambda}$	mc^2	0
after scattering	$\frac{hc}{\lambda'}$	$\frac{h}{\lambda'}$	$\sqrt{m^2c^4 + p'^2c^2}$	p'

Note that momentum is a vector with more than one component!

