Ayres Freitas http://www.pitt.edu/~afreitas/phy1370.html

Homework for week of 08/25/2008

Deadline 09/03/2008

Problem 1: *(1 point)*

Show that the Euler-Lagrange equations in the Lagrange formalism

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_i}, \tag{1}$$

and the canonical equations in the Hamilton formalism

$$\frac{\partial \mathcal{H}}{\partial p_i} = \dot{x}_i, \qquad \frac{\partial \mathcal{H}}{\partial x_i} = -\dot{p}_i, \qquad \mathcal{H} = \sum_i \dot{x}_i p_i - \mathcal{L} \tag{2}$$

are equivalent.

Note: $\mathcal{H} = \mathcal{H}(x_1, x_2, \ldots, p_1, p_2, \ldots)$ and $\mathcal{L} = \mathcal{L}(x_1, x_2, \ldots, \dot{x}_1, \dot{x}_2, \ldots)$ are functions of different variables, so be careful with partial derivatives and use the chain rule where necessary.

Problem 2: Review of basic vector algebra *(3 points)*

Consider an esalistic collision between billard ball A with momentum p_A and billard ball B with momentum p_B. After the collision the momenta are p'_A and p'_B, respectively. The x-, y-, and z-components of the momenta are given by

$$p_A = \begin{pmatrix} -1-\sqrt{2} \\ -1+\sqrt{2} \\ 2 \end{pmatrix}, \qquad p_B = \begin{pmatrix} \sqrt{2} \\ -\sqrt{2} \\ 0 \end{pmatrix}, \qquad p'_A = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \tag{3}$$

a) Calculate the angle between the incoming and outgoing direction of B.

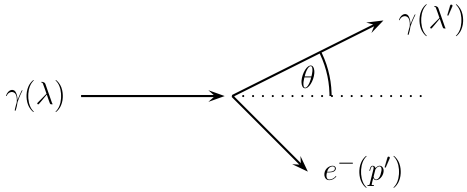
b) Compute $M \cdot p_B$ where

$$M = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 3 & 3 & 1 \end{pmatrix}. \tag{4}$$

What special property does p_B have for M ?

c) Determine all eigenvalues of M.

Problem 3: Compton scattering *(2 points)*



An incident X-ray photon with wavelength λ hits an electron at rest. The two particles scatter elastically, with the outgoing photon having wavelength λ' and the outgoing electron having momentum p'.

Show that

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta), \tag{5}$$

where θ is the angle between incoming and outgoing photon.

Hint: use energy-momentum conservation and the energy-momenum relation from special relativity:

	photon energy	photon momentum	electron energy	electron momentum
before scattering	$\frac{hc}{\lambda}$	$\frac{h}{\lambda}$	mc^2	0
after scattering	$\frac{hc}{\lambda'}$	$\frac{h}{\lambda'}$	$\sqrt{m^2c^4 + p'^2c^2}$	p'

Note that momentum is a vector with more than one component!