Use the simulation software to check the result of Stern-Gerlach experiment.

Note: For all the SGA (Stern-Gerlach apparatus) used in the simulation, the magnetic field gradient is always in the negative direction. The gradient of SGX, SGY and SGZ shown in the simulation have the direction along –x, –y and –z axis. So they will deflect the spin-up state in the upper channel and spin-down state in the lower channel.

Tutorial in class

1. Which one of the following gives the correct relationship between the normalized eigenstates of $\hat{S}_z$ and $\hat{S}_x$?

A. $\left|\uparrow\right\rangle_z = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_x + i\left|\downarrow\right\rangle_x\right)$, $\left|\downarrow\right\rangle_z = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_x - i\left|\downarrow\right\rangle_x\right)$

B. $\left|\uparrow\right\rangle_z = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_x + \left|\downarrow\right\rangle_x\right)$, $\left|\downarrow\right\rangle_z = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_x - \left|\downarrow\right\rangle_x\right)$

C. $\left|\uparrow\right\rangle_z = \frac{1}{\sqrt{2}}\left(a\left|\uparrow\right\rangle_x + b\left|\downarrow\right\rangle_x\right)$, $\left|\downarrow\right\rangle_z = \frac{1}{\sqrt{2}}\left(a\left|\uparrow\right\rangle_x - b\left|\downarrow\right\rangle_x\right)$, where a and b can be any complex numbers that satisfy $|a|^2 + |b|^2 = 1$

D. $\left|\uparrow\right\rangle_z = \left|\downarrow\right\rangle_x$, $\left|\downarrow\right\rangle_z = \left|\uparrow\right\rangle_x$

2. John sends silver atoms in the $\left|\uparrow\right\rangle_z$ state through an SGX-. He places a “down” detector to block some silver atoms and collects the atoms coming out in the “upper channel”. What is the probability of the “down” detector clicking when John sends the silver atoms?

A. 1
B. 0
C. 0.5
D. It is between 0 and 0.5, but the exact probability cannot be inferred from the given information.

Now you can click “File→Load XML→z-up pass SGX-” to check your answers.

3. In the previous experiment, which one of the following normalized spin states has John prepared in the "upper channel"?

A. $|\uparrow\rangle_z$

B. $|\uparrow\rangle_x$

C. $\frac{1}{2}|\uparrow\rangle_z$

D. $\frac{1}{\sqrt{2}}|\uparrow\rangle_z$

Now you can click "File→Load XML→z-up pass SGX-1" and "File→Load XML→z-up pass SGX-2" to check your answers. In the former file, the atoms prepared in the "upper channel" passed a SGZ-. In the latter one, the atoms prepared in the upper channel passed a SGX-.

4. If John measures $\hat{S}_x$ for the atoms he prepared in the "upper channel", what is the probability of measuring $+\frac{\hbar}{2}$?

A. 1
B. 0
C. 0.5
D. It is between 0 and 0.5, but the exact probability cannot be inferred from the given information.

Now you can use "File→Load XML→z-up pass SGX-2" to check your answers.

5. Which one of the following gives the correct relationship between the normalized eigenstates of $\hat{S}_z$ and $\hat{S}_y$?

A. $|\uparrow\rangle_z = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_y + i|\downarrow\rangle_y\right)$, $|\downarrow\rangle_z = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_y - i|\downarrow\rangle_y\right)$

B. $|\uparrow\rangle_z = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_y + |\downarrow\rangle_y\right)$, $|\downarrow\rangle_z = \frac{-i}{\sqrt{2}}\left(|\uparrow\rangle_y - |\downarrow\rangle_y\right)$

C. $|\uparrow\rangle_z = \frac{1}{\sqrt{2}}\left(a|\uparrow\rangle_x + b|\downarrow\rangle_x\right)$, $|\downarrow\rangle_z = \frac{1}{\sqrt{2}}\left(a|\uparrow\rangle_x - b|\downarrow\rangle_x\right)$, where a and b can be any complex numbers that satisfy $|a|^2 + |b|^2 = 1$

D. $|\uparrow\rangle_z = |\downarrow\rangle_x$, $|\downarrow\rangle_z = |\uparrow\rangle_x$

6. Check that ${}_z\langle\uparrow|\downarrow\rangle_z = 0$ by expressing it in the y-basis above.

7. John sends silver atoms in the $|\uparrow\rangle_z$ state through an SGY-. He places a “down” detector to block some silver atoms and collects the atoms coming out in the “upper channel”. What is the probability of the “down” detector clicking when John sends the silver atoms?

A. 1
B. 0
C. 0.5
D. It is between 0 and 0.5, but the exact probability cannot be inferred from the given information.

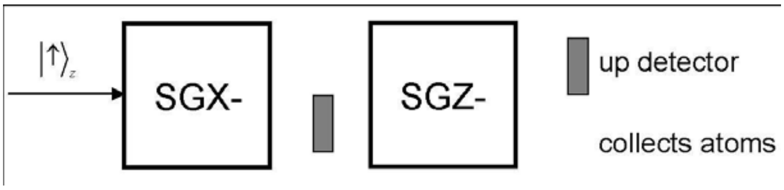
Now you can click “File→Load XML→z-up pass SGY-” to check your answers.

8. In the previous experiment, which one of the following normalized spin states has John prepared in the “upper channel”?

A. $|\uparrow\rangle_z$
B. $|\uparrow\rangle_y$
C. $\frac{1}{2}|\uparrow\rangle_y$
D. John has not prepared anything. Everything gets blocked by the down “down” detector.

Now you can click “File→Load XML→z-up pass SGY-1” and “File→Load XML→z-up pass SGY-2” to check your answers. In the former file, the atoms prepared in the “upper channel” passed a SGZ-. In the latter one, the atoms prepared in the upper channel passed a SGY-.

9. You send silver atoms in an initial spin state $\left|\uparrow\right\rangle_z$ one at a time through two SGAs with magnetic field gradients as shown below. Suitable detectors are placed as shown. One detector is between the two SGAs and the other after both SGAs. What is the probability that a given single atom will cause the “up” detector to click after passing through this system of two SGAs?



A. 1
B. 0
C. 0.5
D. 0.25

Now you can click “File→Load XML→z-up pass SGX-1” to check your answer.

10. In the previous experiment (problem 9), you collect the silver atoms that are not blocked at the end after they have passed through both SGAs. Which one of the following is the state of the silver atom you collect?

A. $\left|\uparrow\right\rangle_z$

B. $\left|\downarrow\right\rangle_z$

C. $\left|\downarrow\right\rangle_x$

D. You do not collect anything because all atoms passing through the second SGA are blocked by the detector

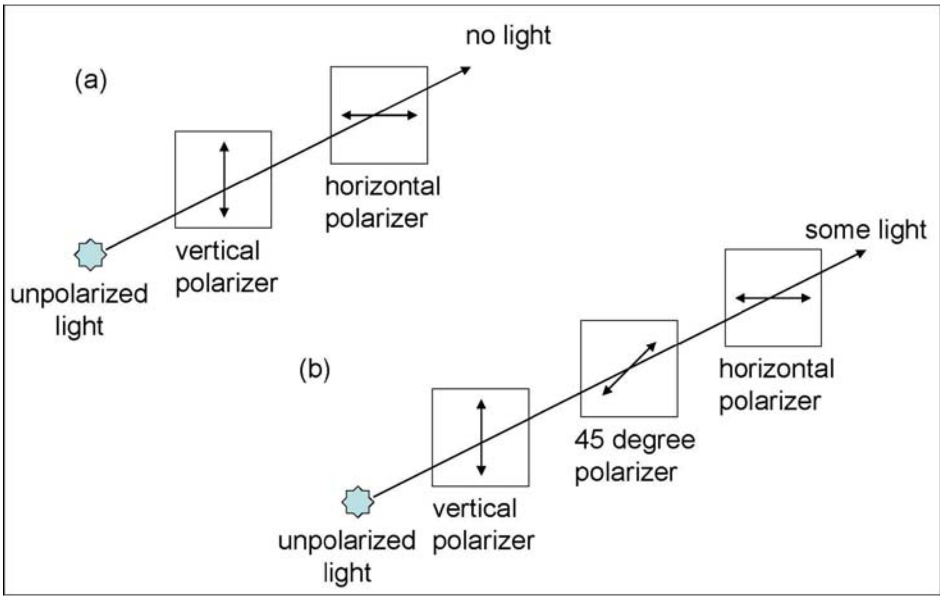
Now you can click “File→Load XML→z-up pass SGX- and SGZ-” to check your answers.

11. Consider the following conversation between Andy and Caroline:

★Andy: There must be something wrong with the answer to the previous question. How can the $\left|\uparrow\right\rangle_z$ that we inputted give $\left|\downarrow\right\rangle_z$ on the way out?

★Caroline: I disagree. If you let atoms in the state $\left|\uparrow\right\rangle_z$ pass through the SGZ only, you will never obtain $\left|\downarrow\right\rangle_z$ on the way out. However, $\left|\downarrow\right\rangle_z$ is obtained in the above experiment because we have inserted an SGX at an intermediate stage. Think of the analogy with vertically polarized light passing directly through a horizontal polarizer vs. passing first through a polarizer at 45^o followed by a horizontal polarizer. There is no light at the output if vertically polarized light passes directly through a horizontal polarizer. On the other hand, if the polarizer at 45^o is present, light becomes polarized at 45^o after the 45^o polarizer which is a linear superposition of horizontal and vertical polarization. Therefore, some light comes out through the horizontal polarizer placed after the 45^o polarizer.

Do you agree with Caroline? Explain whether the spin angular momentum of the atoms remain conserved or not in the previous experiment (problem 9).



12. or the problem above (problem 9), write down the spin wave function $|\uparrow\rangle_z$ <u>before</u> passing through the SGX. Choose a basis that is most suitable for determining the result of the SGX.

13. For the problem above (problem 9), the "down" detector between SGX and SGZ will collapse the state of the silver atoms. If the detector clicks, the atom gets absorbed by the detector. If the detector does not click, write down the spin wave function after passing through the SGX (between SGX and SGZ) right before entering the SGZ.

14. Use your result for the previous part to find the probability of the "up" detector clicking and confirm your response to the multiple choice question earlier.

15. Consider the following conversation between Andy and Caroline:

★Andy: Is an eigenstate of any one component of spin, say $|\uparrow\rangle_y$ for $\hat{S}_y$, a superposition of the eigenstates of any of the other two components, say $\hat{S}_z$ or $\hat{S}_x$, with equal weight?

★Caroline: Yes. There may be a phase factor such as "i" (where $\pm i = \sqrt{-1}$) when you write $|\uparrow\rangle_y$ in terms of the superposition of $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ but the probability is the same for both eigenstates of $\hat{S}_z$.

Do you agree with Caroline? Explain.

Question 16 and 17 relate to the simulation "File→Load XML→unknown state".

16. Write down at least 3 different possible spin states of the incoming particles that will show the behavior seen in the simulation. The incoming particles do not necessarily have identical spin states. Explain your reason.

17. Choose two of the different possible spin states you predicted for the simulation you saw. Now come up with some experiments using SGA that would distinguish between the two possible spin states. You can choose one or more SGA to find out which of the two spin states it is.