Game Theory Principles VII

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The Extensive Form Representation of a Game

We want to express more explicitly the role of **information** in games. This involves:

1. information that is **private**ly held by players at the beginning of a game (e.g. their valuation for an object in an auction)
2. the lack of information about **other player's moves** (e.g. when actions are taken simultaneously)

An Example: The Policy Advice Game

1. There are two possible state of the world, s and t.
2. Nature "chooses" state t with probability $2/3$.
3. The expert privately learns the state of the world, s or t (which we will refer to as his type).
4. The expert sends a message m or n to the decision maker.
5. Having observed the expert's message, the decision maker chooses either action a or b.

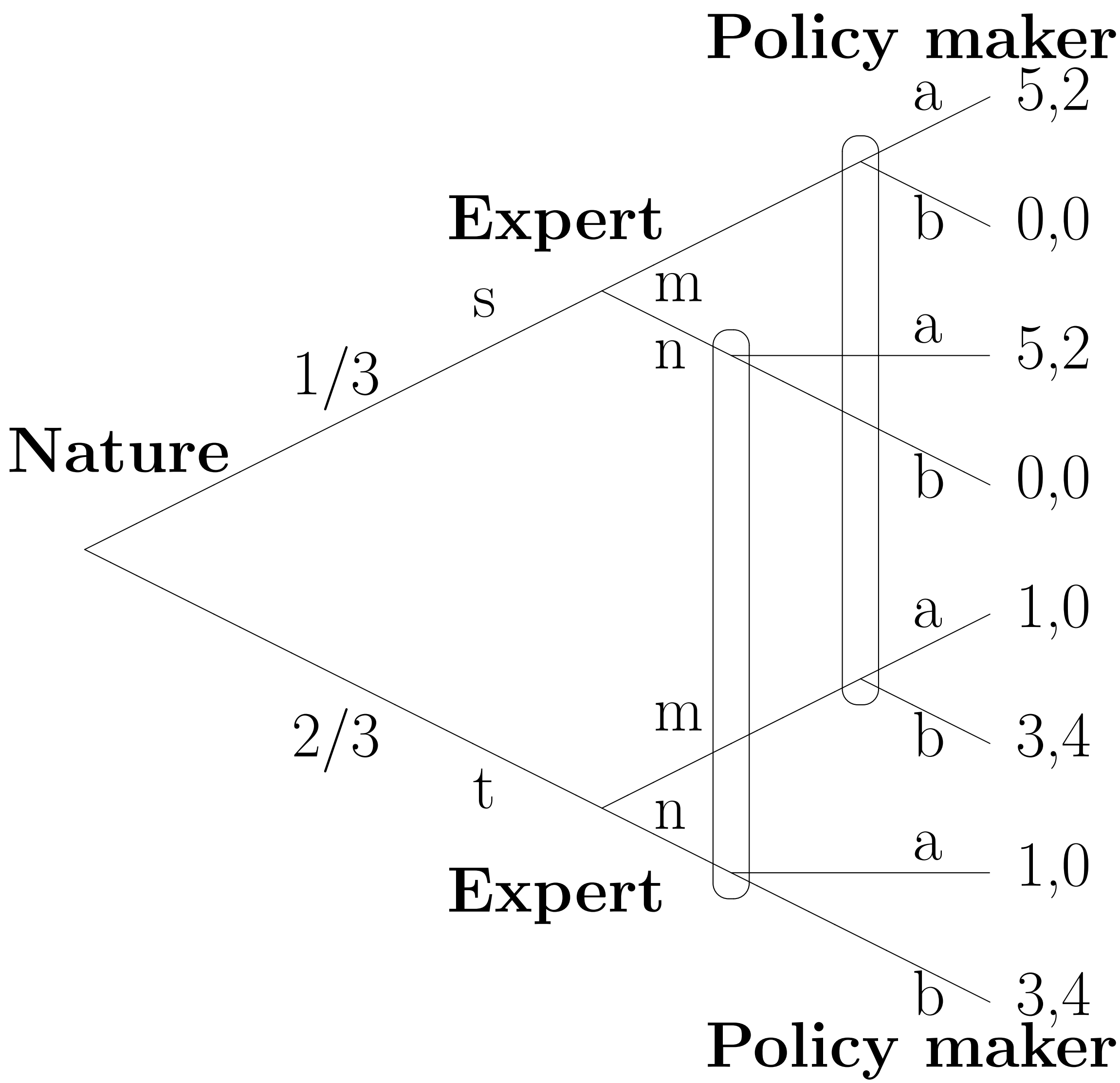
6. The payoffs to both players are given by the following table:

	a	b
s	$5, 2$	$0, 0$
t	$1, 0$	$3, 4$

We want a game tree for the policy advice game that captures

1. nature's choice of the state of the world,
2. the expert's ability to distinguish the two states of the world,
3. the policy maker's inability to distinguish the two states of the world, and
4. the policy maker's ability to distinguish the expert's messages.

Game tree for the policy advice game



Game VII-1

“Reading” the game tree from left to right:

1. At the **initial node** nature chooses either state s or state t.
2. **Nature’s choices are** governed by fixed **probabilities**.
3. The **probability of state** s is $1/3$ and the **probability of state** t is $2/3$**.**

4. The expert learns the state, s or t. This is expressed by the fact that the expert's **information sets** each contain only a single **decision node**.
5. At each of his two information sets the expert has a choice between two **actions**: sending message m or message n. (Notice that we use the word "action" here in a generic sense; any decision made by any player in the game is referred to as an action.)

6. The policy maker observes the message sent by the expert but not the **type** of the expert who sent the message. This is expressed by including the decision nodes of the expert that belong to different types in the same **information set**.
7. The policy maker's information sets are indicated by oval. The decision nodes belonging to an information set are the ones contained in the corresponding oval.
8. The policy maker has two information sets that are distinguished by the messages used to reach them.

9. At each of his information sets, the policy maker chooses among two actions, action a and action b.
10. The expert's type and the policy maker's action determine both players' payoffs.
11. We will adhere to the convention that for all the indicated payoff pairs, the first entry is the expert's payoff and the second entry is the policy maker's payoff.

Game trees in general

1. Each game tree has a single **initial node**.
2. Each game tree has one or more **decision nodes**.
3. There are multiple **branches** emanating from each decision node.
4. Each branch leads to another node, either a decision node or a terminal **terminal node**.
5. Each game tree has multiple terminal nodes.
6. For each terminal node, there is one and only one set of branches that connect that terminal node to the initial node.

7. Each terminal node is assigned a **payoff** for each of the players in the game.
8. Each decision node is assigned a **player** who moves at that node.
9. The branches that emanate from a decision node correspond to the **choices** that are available to the player who moves at that node.

10. Each decision node belongs to the **information set** of some player moving at that node.
11. A player moving at an **information set** cannot distinguish her decision nodes belonging to that set.
12. Assumption of **perfect recall**: Unless explicitly state otherwise, we will only place information sets in such a way that players do not forget information they had earlier in the game.
13. Each decision node belongs to one and only one information set.

Strategies and the extensive form representation

1. Strategies are exactly what they used to be before, i.e. functions from histories into actions.
2. Notice that we can identify histories with information sets.
3. Therefore, equivalently we can say that a strategy for a player is an assignment of actions to each of her/his information sets.

4. For example, the policy maker in our policy advice game has two information sets, one corresponding to (receiving) the message m and the other to message n; at each information set he has two actions a and b. Hence, he has four strategies.

The extensive form and the strategic form representation of a game

1. Since strategies are exactly what they used to be before, we can use the payoffs in the **extensive form representation** to calculate payoffs for all different strategy combinations.
2. This gives us the **strategic form representation** of a game.
3. Notice that for any represented in extensive form there is a unique way of representing that same game in strategic form.

Strategies in the policy advice game:

1. Let (m, n) denote the expert's strategy "send message m if the state is s and send message n if the state is t."

2. Use similar notation for the remaining strategies of the expert.

3. Let (a, b) denote the policy maker's strategy "use action a if the message is m and use action b if the message is n."

4. Use similar notation for the remaining strategies of the policy maker.

The policy advice game in strategic form:

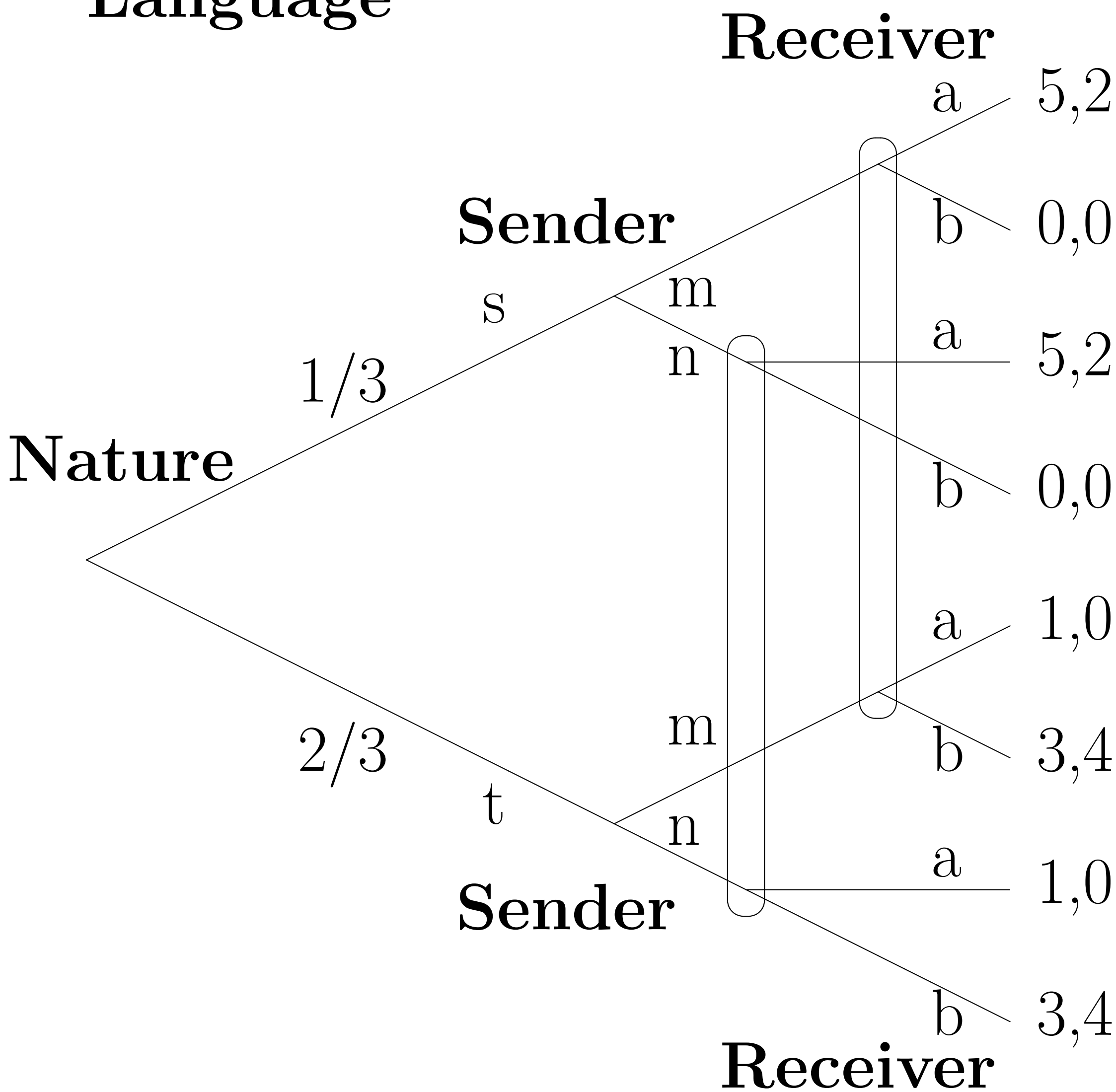
Policy maker

Expert

	(a,a)	(a,b)	(b,a)	(b,b)
(m,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$2, \frac{8}{3}$
(m,n)	$\frac{7}{3}, \frac{2}{3}$	$\frac{11}{3}, \frac{10}{3}$	$\frac{2}{3}, 0$	$2, \frac{8}{3}$
(n,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{2}{3}, 0$	$\frac{11}{3}, \frac{10}{3}$	$2, \frac{8}{3}$
(n,n)	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$

GAME VII-2

Same game, different story: Conventions in Language



Game VII-3

The language game in strategic form:

Receiver

Sender

	(a,a)	(a,b)	(b,a)	(b,b)
(m,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$2, \frac{8}{3}$
(m,n)	$\frac{7}{3}, \frac{2}{3}$	$\frac{11}{3}, \frac{10}{3}$	$\frac{2}{3}, 0$	$2, \frac{8}{3}$
(n,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{2}{3}, 0$	$\frac{11}{3}, \frac{10}{3}$	$2, \frac{8}{3}$
(n,n)	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$

Game VII-4

Let's say that sender and receiver have a **shared language** if in equilibrium different messages have different states of the world as their referents.

An equilibrium without a shared language:

	Receiver			
Sender	(a,a)	(a,b)	(b,a)	(b,b)
(m,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\mathbf{2, \frac{8}{3}}$
(m,n)	$\frac{7}{3}, \frac{2}{3}$	$\frac{11}{3}, \frac{10}{3}$	$\frac{2}{3}, 0$	$2, \frac{8}{3}$
(n,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{2}{3}, 0$	$\frac{11}{3}, \frac{10}{3}$	$2, \frac{8}{3}$
(n,n)	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$

GAME VII-4

Another equilibrium without a shared language:

Sender \ Receiver	(a,a)	(a,b)	(b,a)	(b,b)
(m,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$2, \frac{8}{3}$
(m,n)	$\frac{7}{3}, \frac{2}{3}$	$\frac{11}{3}, \frac{10}{3}$	$\frac{2}{3}, 0$	$2, \frac{8}{3}$
(n,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{2}{3}, 0$	$\frac{11}{3}, \frac{10}{3}$	$2, \frac{8}{3}$
(n,n)	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\frac{7}{3}, \frac{2}{3}$	**$2, \frac{8}{3}$**

GAME VII-4

And one more equilibrium without a shared language (in this equilibrium think of the sender as randomizing and picking (m, n) and (n, m) with equal probability):

Sender		Receiver (a,a)	(a,b)	(b,a)	(b,b)
	(m,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$2, \frac{8}{3}$
	(m,n)	$\frac{7}{3}, \frac{2}{3}$	$\frac{11}{3}, \frac{10}{3}$	$\frac{2}{3}, 0$	$\mathbf{2, \frac{8}{3}}$
	(n,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{2}{3}, 0$	$\frac{11}{3}, \frac{10}{3}$	$\mathbf{2, \frac{8}{3}}$
	(n,n)	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$

GAME VII-4

An equilibrium with a shared language:

	Receiver			
Sender	(a,a)	(a,b)	(b,a)	(b,b)
(m,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$2, \frac{8}{3}$
(m,n)	$\frac{7}{3}, \frac{2}{3}$	$\mathbf{\frac{11}{3}, \frac{10}{3}}$	$\frac{2}{3}, 0$	$2, \frac{8}{3}$
(n,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{2}{3}, 0$	$\frac{11}{3}, \frac{10}{3}$	$2, \frac{8}{3}$
(n,n)	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$

Game VII-4

Another equilibrium with a shared language:

		Receiver			
		(a,a)	(a,b)	(b,a)	(b,b)
	(m,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$2, \frac{8}{3}$
	(m,n)	$\frac{7}{3}, \frac{2}{3}$	$\frac{11}{3}, \frac{10}{3}$	$\frac{2}{3}, 0$	$2, \frac{8}{3}$
Sender	(n,m)	$\frac{7}{3}, \frac{2}{3}$	$\frac{2}{3}, 0$	$\mathbf{\frac{11}{3}, \frac{10}{3}}$	$2, \frac{8}{3}$
	(n,n)	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$	$\frac{7}{3}, \frac{2}{3}$	$2, \frac{8}{3}$

Game VII-4

Conventions in the language game

1. The language game has multiple Nash equilibria.
2. Some of these equilibria correspond to conventions with a shared language.
3. Not all equilibria correspond to conventions with a shared language.

The emergence of conventions in the language game

Think of a population of players, some senders and some receivers, who repeatedly interact and play the language game.

Recall our simple adjustment rule:

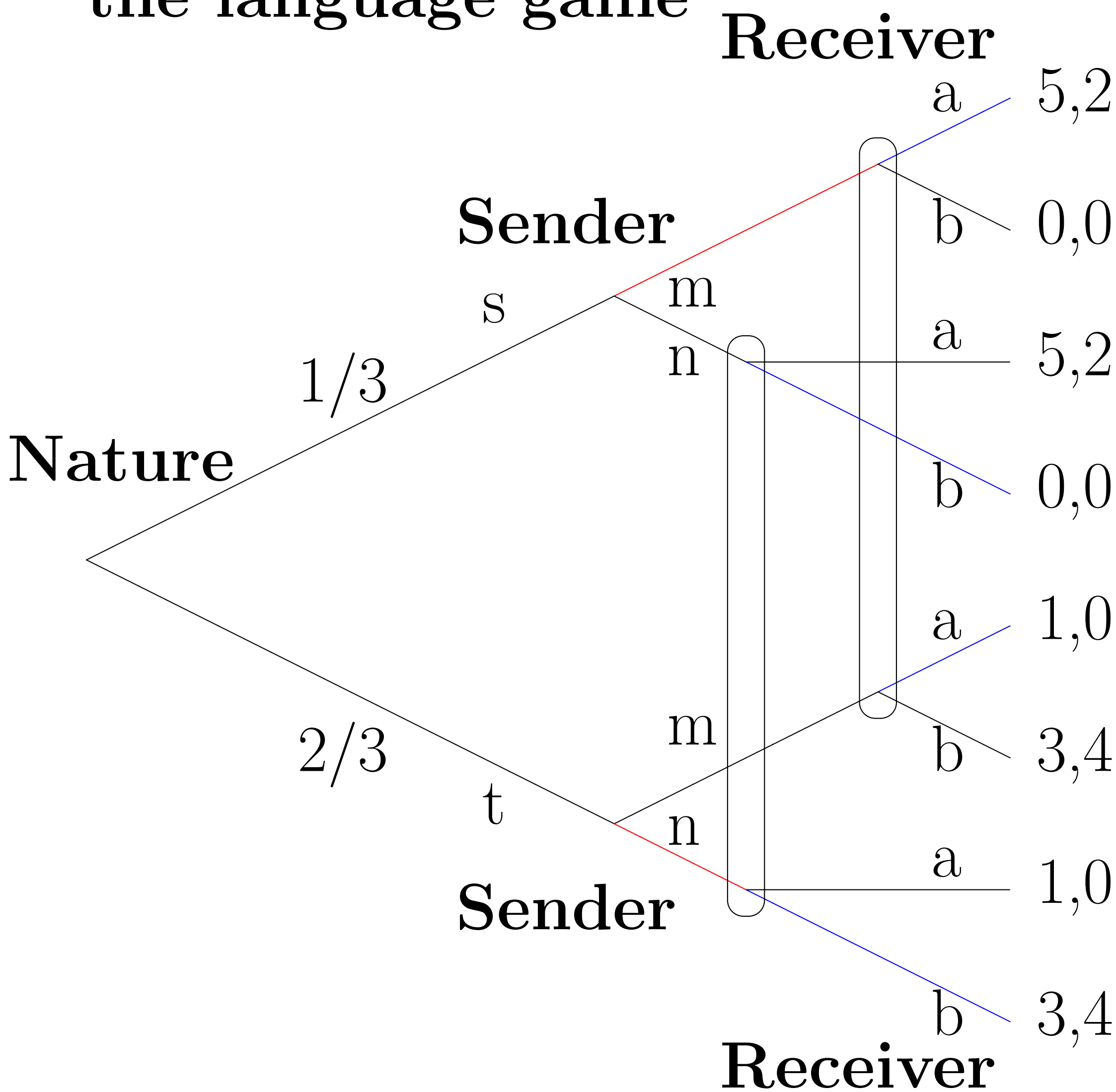
1. In each period one of the players is randomly selected.
2. The selected player observes the **current strategy choices** of all other players.
3. The selected player adopts a strategy that is a best response to the distribution of strategies among all players

1. Take as the starting point a state in which all senders use the strategy (m, m) and all receivers use the strategy (b, b), i.e. a state corresponding to an equilibrium without a shared language.
2. Notice that (b, a) is a best response for receivers to (m, m).
3. There is positive probability that for a long time only receivers are selected to adjust their strategies and that all those receivers switch to (b, a).

4. Hence, there is positive probability that over time the population reaches the disequilibrium state where all senders use the strategy (m, m) and all receivers use the strategy (a, b).
5. There is positive probability that now for a long time only receivers are chose to adjust their strategy.
6. These receivers would all switch to (m, n).

7. Hence, there is positive probability that over time we **reach a convention with a shared language**.

A shared language in the extensive form of the language game



Game VII-5