Game Theory Principles VI

Andreas Blume
Department of Economics
University of Pittsburgh
Pittsburgh, PA 15260

GAME THEORY IN PHILOSOPHY: Ethics, norms, the *stag hunt game*, principles of justice, `http://plato.stanford.edu/entries/game-ethics/`,

knowledge, common knowledge, awareness.

Game theory questions with a philosophy flavor:

1. What is the origin of conventions (e.g. money, property rights, language)?
2. Does game theory have anything to say about principles of justice?
3. What is the relationship between individual and collective rationality?

4. What can we say about behavior in a game based only on the assumption that agents are rational?
5. What if in addition we assume that agents know that other agents are rational?
6. What if in addition we assume that everyone knows that everyone knows that everyone is rational?
7. What if we adopt the assumption of common knowledge of rationality?

8. How should we represent knowledge in a game?
9. How should we represent belief in a game?
10. How should we deal with the possibility of some agents in a game being unaware of some aspects of the game?

Convention

For David Lewis in his book *Convention*, a convention is essentially a Nash equilibrium in a coordination game, i.e. game in which there are multiple Nash equilibria and agents' interests (largely) coincide.

According to David Lewis: *We acquire a general belief, unrestricted as to time, that members of a certain population conform to a certain regularity in a certain kind of recurring coordination problem for the sake of coordination. ... As long as uniform conformity is a coordination equilibrium, so that each wants to conform conditionally upon conformity by the others, conforming action produces expectation of conforming action and expectation of conforming action produces conforming action. This is the phenomenon I call convention.*

Examples of Coordination Problems

The Coffee-Shop Game

1. Alpha and Omega intend to meet in a coffee shop.
2. There are two possible choices: Lazy Latte and Busy Brew.
3. Both prefer meeting to not meeting.
4. Neither of them cares where to meet.

		Omega	
		Lazy Latte	Busy Brew
Alpha	Lazy Latte	(1,1)	(0,0)
	Busy Brew	(0,0)	(1,1)

Game VI-1

1. The coffee-shop game (Game VI-1) has multiple Nash equilibria.
2. The pure strategy equilibria are **strict**, i.e. the equilibrium strategies are unique best replies to each other.
3. The pure strategy equilibria are equally attractive.
4. If Alpha and Omega decide independently which coffee shop to go to, which principle might guide their decisions?

Hume's Row Boat

1. Two rowers row a boat.
2. They intend to move in a straight line.
3. I order to move in a straight line, they need to synchronize their rowing.
4. They do not care about a specific speed.

Omega

		vigorously	leisurely	steadily
	vigorously	1,1	0,0	0,0
Alpha	leisurely	0,0	1,1	0,0
	steadily	0,0	0,0	1,1

GAME VI-2

1. The same comments that we made regarding the coffee-shop game apply here as well.
2. Since there are more choices, the coordination problem appears to be more difficult.
3. The “real” version of the problem appears to be even more difficult when we take into account that there is a whole continuum of different speeds and that to each speed corresponds a strict Nash equilibrium.

Rousseau's Stag Hunt Game

Rousseau writes in *A Discourse on Inequality:*

"If it was a matter of hunting a deer, everyone well realized that he must remain faithful to his post; but if a hare happened to pass within reach of one of them, we cannot doubt that he would have gone off in pursuit of it without scruple..."

(Puzzle: How did the stag become a deer?)

		Neanderthal Man	
		Hunt Stag	Hunt Hare
CroMagnon Woman	Hunt Stag	(9,9)	(0,7)
	Hunt Hare	(7,0)	(7,7)

Game VI-3

1. Like the coffee-shop game and Hume's row boat, Rousseau's stag hunt has multiple strict equilibria.
2. Unlike in the coffee shop game and in Hume's row boat, there is a unique commonly preferred equilibrium in the stag hunt game.
3. Unlike in the coffee shop game and in Hume's row boat, one of the strict equilibria is riskier than the other strict equilibrium in the stag-hunt game.
4. Unlike in the coffee shop game and in Hume's row boat, there is a **tension between** concerns about **optimality and** about **risk** in the stag-hunt game.

Keynesian Demand Failure

One's level of activity in an economy may be a function of one's expectation of the level of activity of other actors in the economy. This raises the possibility that a stock market crash or a recession may be the result of a peculiar coincidence of beliefs that agents hold about each other. A related sentiment is expressed in the following quote:

"Professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view.

It is not a case of choosing those which, to the best of ones judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees . . ."

J.M. Keynes, The General Theory of Employment, Interest and Money, 1936.

John Bryant has formalized a simple game that captures the link between strategic interdependence and Keynesian demand failure.

We will look at two versions of related games that have been experimentally studied by John Van Huyck, Ray Battalio and various coauthors.

The **median** of a sample is its midpoint, such that 50% of the observations have values at or above the median and 50% of the observations have values at or below the median.
(With an even number of observations, it is customary to take the mean of the two middle values.)

The Median Game

1. There is a group of players (for simplicity, make it an odd number of players).
2. All players in the group simultaneously choose effort levels.
3. Each player's payoff depends on both his/her effort and the median effort of other players.
4. The payoff structure encourages **conformity**: Everyone wants to be as close as possible to the median.
5. **Effort levels are ranked**: Conditionally on everyone choosing the same effort level, everyone prefers higher effort.

The following payoff matrix records your payoff as a function of both your own effort and the group median effort.

The Median Effort

Your Effort	7	6	5	4	3	2	1
7	1.30	1.15	0.90	0.55	0.10	-.45	-1.10
6	1.25	1.20	1.05	0.80	0.45	0.00	-.55
5	1.10	1.15	1.10	0.95	0.70	0.35	-.10
4	0.85	1.00	1.05	1.00	0.85	0.60	0.25
3	0.50	0.75	0.90	0.95	0.90	0.75	0.50
2	0.05	0.40	0.65	0.80	0.85	0.80	0.65
1	-.50	-.05	0.30	0.55	0.70	0.75	0.70

GAME VI-4

Equilibria in the median game

The Median Effort

Your Effort	7	6	5	4	3	2	1
7	**1.30**	1.15	0.90	0.55	0.10	-.45	-1.10
6	1.25	**1.20**	1.05	0.80	0.45	0.00	-.55
5	1.10	1.15	**1.10**	0.95	0.70	0.35	-.10
4	0.85	1.00	1.05	**1.00**	0.85	0.60	0.25
3	0.50	0.75	0.90	0.95	**0.90**	0.75	0.50
2	0.05	0.40	0.65	0.80	0.85	**0.80**	0.65
1	-.50	-.05	0.30	0.55	0.70	0.75	**0.70**

Game VI-4

1. The median game has multiple strict equilibria.
2. The strict equilibria are Pareto ranked.
3. Notice that some actions are safer than others: For example, effort level 3 guarantees a payoff of at least 0.50, whereas the payoff associated with the unique efficient Nash equilibrium could be as low as -1.10.

4. Without prior experience with the game and absent communication, players are likely to face **strategic uncertainty**: They will be unable to predict exactly the actions of other players.

5. With strategic uncertainty players may want to avoid actions that appear to carry too much **risk**.

6. Therefore, it is likely that initially players will choose other than the highest effort level.
7. Hence, the **initial median** is likely to be less than the highest effort level.
8. If the game is repeated, the initial median becomes **a natural reference point for future effort choices**.

The Minimum Game

1. There is a group of players.
2. All players in the group simultaneously choose effort levels.
3. Each player's payoff depends on both his/her effort and the minimum effort of other players.
4. The payoff structure encourages **conformity**: Everyone wants to be as close as possible to the minimum of other players' choices.
5. **Effort levels are ranked**: Conditionally on everyone choosing the same effort level, everyone prefers higher effort.

The Minimum Effort

Your Effort	7	6	5	4	3	2	1
7	1.30	1.10	0.90	0.70	0.50	0.30	0.10
6		1.20	1.00	0.80	0.60	0.40	0.20
5			1.10	0.90	0.70	0.50	0.30
4				1.00	0.80	0.60	0.40
3					0.90	0.70	0.50
2						0.80	0.60
1							0.70

Game VI-5

Equilibria in the minimum game

The Minimum Effort

Your Effort	7	6	5	4	3	2	1
7	**1.30**	1.10	0.90	0.70	0.50	0.30	0.10
6		**1.20**	1.00	0.80	0.60	0.40	0.20
5			**1.10**	0.90	0.70	0.50	0.30
4				**1.00**	0.80	0.60	0.40
3					**0.90**	0.70	0.50
2						**0.80**	0.60
1							**0.70**

GAME VI-5

1. The minimum game has multiple strict equilibria.
2. The strict equilibria are Pareto ranked.
3. Notice that some actions are safer than others: The minimum effort guarantees a payoff of .70, whereas no other effort level guarantees an equally high payoff.

4. Without prior experience with the game and absent communication, players are likely to face **strategic uncertainty**: They will be unable to predict exactly the actions of other players.

5. With strategic uncertainty players may want to avoid actions that appear to carry too much **risk**.

6. Therefore, it is likely that initially players will choose other than the highest effort level.
7. Hence, there is likely to be considerable dispersion in individual effort levels.
8. As a consequence of dispersion, the minimum chosen effort level is likely to be low.
9. If the game is repeated, persistent dispersion is likely to exert pressure on effort levels to fall.
10. There appears to be a good chance that over time a **minimum effort level convention** will be established.

For experimental evidence on both the median game and the minimum game, consult the paper by Blume and Ortmann in the *Journal of Economic Theory* and the references cited therein.

Simple Dynamics in Games

1. Recall that in Lewis's discussion of conventions it is important that interactions occur in a **population** and are **recurrent**.

2. Let's construct a **simple dynamic process** by which members of a population of players adjust their strategies in a game.

3. We are interested in whether such a dynamic process might **converge** to a convention over time.

For simplicity, we restrict attention to symmetric games (in which all players have the same set of available strategies).

The Adjustment Rule

1. In each period one of the players is randomly selected.
2. The selected player observes the **current strategy choices** of all other players.
3. The selected player adopts a strategy that is a best response to the distribution of strategies among all players (for technical reasons, it is convenient to have a player play a best response to the distribution of strategies of all players rather than only of other players).

Applying the adjustment rule to the stag-hunt game:

For simplicity (and without loss of generality), consider a population with 10 members.

1. If currently all ten members of the population hunt stag, the randomly selected individual has an expected payoff of 9 from hunting stag and of 7 from hunting hare against the population strategy. His unique best reply is to continue to hunt stag.

2. Suppose that currently 9 members of the population hunt stag and 1 hunts hare. Therefore the expected payoff in the population from hunting stag equals:

$$\frac{9}{10} \times 9 + \frac{1}{10} \times 0 = 8.1.$$

Similarly, his expected payoff from hunting hare equals:

$$\frac{9}{10} \times 7 + \frac{1}{10} \times 7 = 7.$$

Hence, any randomly selected individual will choose to hunt stag.

3. Suppose that currently 8 members of the population hunt stag and 2 hunt hare. Therefore the expected payoff in the population from hunting stag equals:

$$\frac{8}{10} \times 9 + \frac{2}{10} \times 0 = 7.2.$$

Similarly, his expected payoff from hunting hare equals:

$$\frac{8}{10} \times 7 + \frac{2}{10} \times 7 = 7.$$

Hence, any randomly selected individual will choose to hunt stag.

4. Suppose that currently 7 members of the population hunt stag and 3 hunt hare. Therefore the expected payoff in the population from hunting stag equals:

$$\frac{7}{10} \times 9 + \frac{3}{10} \times 0 = 6.3.$$

Similarly, his expected payoff from hunting hare equals:

$$\frac{7}{10} \times 7 + \frac{3}{10} \times 7 = 7.$$

Hence, any randomly selected individual will choose to hunt hare.

5. Suppose that currently 6 members of the population hunt stag and 4 hunt hare. Therefore the expected payoff in the population from hunting stag equals:

$$\frac{6}{10} \times 9 + \frac{4}{10} \times 0 = 5.4.$$

Similarly, his expected payoff from hunting hare equals:

$$\frac{6}{10} \times 7 + \frac{4}{10} \times 7 = 7.$$

Hence, any randomly selected individual will choose to hunt hare.

6. Suppose that currently 5 members of the population hunt stag and 5 hunt hare. Therefore the expected payoff in the population from hunting stag equals:

$$\frac{5}{10} \times 9 + \frac{5}{10} \times 0 = 4.5.$$

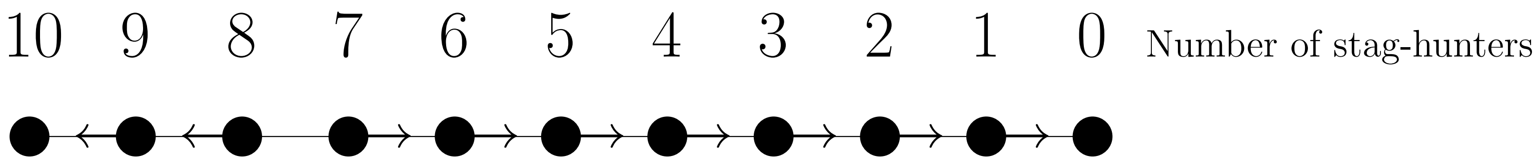
Similarly, his expected payoff from hunting hare equals:

$$\frac{5}{10} \times 7 + \frac{5}{10} \times 7 = 7.$$

Hence, any randomly selected individual will choose to hunt hare.

7. ...

A graphical representation of our dynamic in the stag-hunt game:



DYNAMICS IN THE STAG-HUNT GAME

1. The population can be in eleven different states.
2. In three of these state (states 10, 9 and 8) the population will move toward the stag-hunting convention.
3. In the remaining states (states 7, 6, . . .) the population will move toward the hare hunting convention.

The set of states from which the population ultimately reaches the stag-hunting convention (i.e. **states 10, 9 and 8**) are referred to as the **basin of attraction** of the **stag-hunting convention**.

The set of states from which the population ultimately reaches the hare-hunting convention (i.e. **states 7, 6, 5,** ...) are referred to as the **basin of attraction** of the **hare-hunting convention**.

Notice that the basin of attraction of the hare-hunting convention is larger (i.e. includes more states) than the basin of attraction of the stag-hunting convention.

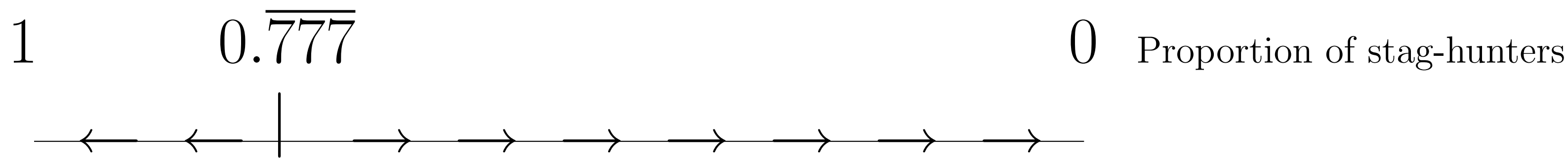
Therefore, if the initial population state is randomly chosen, the population is more likely to end up in the hare-hunting convention than in the stag-hunting convention.

The conclusions we have reached for our dynamic in a population with 10 members extends to populations of any size:

1. If the fraction of stag hunters in the population exceeds $\frac{7}{9} = 0.\overline{777}$, the fraction of stag hunters in the population will increase until there are only stag hunters.

2. If the fraction of stag hunters in the population is less than $\frac{7}{9} = 0.\overline{777}$, the fraction of stag hunters in the population will fall until there are only hare hunters.

Population dynamics in the stag-hunt game for arbitrary population sizes:



DYNAMICS IN THE STAG-HUNT GAME

1. Notice that $0.\overline{777}$ is the probability of the stag-hunting strategy in the mixed strategy equilibrium of the stag-hunt game.
2. Thus, the mixed-strategy equilibrium separates the basins of attraction of the stag-hunt convention and the hare-hunt convention.

3. The **stag-hunt convention** is **locally stable;** i.e., following a sufficiently small displacement from the stag-hunt convention, the population will return to the stag-hunt convention.

4. The **hare-hunt convention** is **locally stable;** i.e., following a sufficiently small displacement from the hare-hunt convention, the population will return to the hare-hunt convention.

5. A population state corresponding to the **mixed-strategy equilibrium** is **not locally stable.**

Dynamics in the Minimum Game

1. Our dynamic can equally well be used to analyze the minimum game.
2. Here the state pace has six dimensions, one for each strategy, minus one because the strategy proportions have to sum to one.

3. Notice that the **basin of attraction of the unique efficient equilibrium is extremely small**: any state in which at least one individual uses an action other than action 7 will not converge to the efficient equilibrium.
4. In contrast, the **basin of attraction of the least efficient pure strategy equilibrium is quite large:** as long as at least one individual in the population uses action 1, all members of the population will eventually adopt action 1.

A Stochastic Adjustment Rule

There is a small group of "random players" who sometime make random choices. All other players use the adjustment rule:

1. In each period one of the players is randomly selected.
2. The selected player observes the **current strategy choices** of all other players.
3. The selected player adopts a strategy that is **a best response** to the distribution of strategies among all players.

With the stochastic dynamic in the minimum game the **population state converges to everyone using the minimum action** regardless of the initial population state.