Game Theory Principles II

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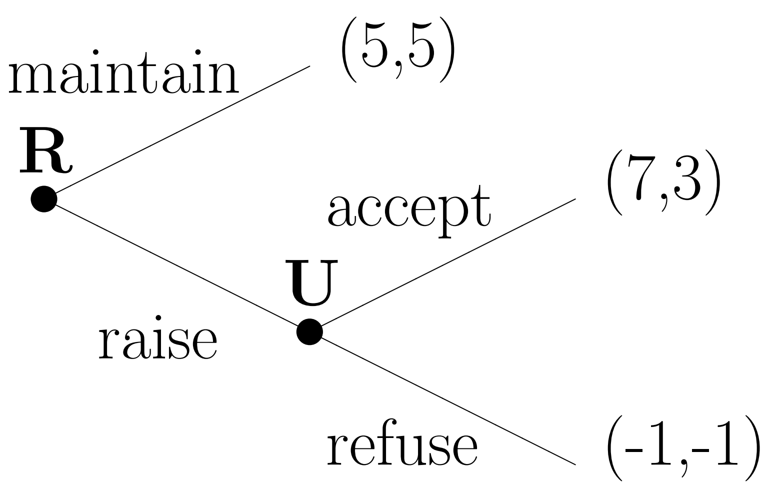
Game theory in economics 1: Adam Smith, gains from trade vs. the prisoners' dilemma, games that promote efficient outcomes vs. games that discourage efficiency. Concepts: Rationality, efficiency, utility maximization, strategic domination, best response, Nash equilibrium. Tools: game matrices.

Recall that in the Russia-Ukraine game, Russia has two strategies, **maintain** and **raise**. Ukraine also has two strategies, **accept** and **refuse**. Notice that once we choose a strategy for each player, e.g. **raise** for Russia and **accept** for Ukraine, the resulting strategy pair (here (**raise**, **accept**)) determines a payoff for each player (here **(7,3)**).

(R,U)



Game I-3

Payoffs for all four strategy pairs of the Russia-Ukraine game, Game I-3:

1. The strategy pair (raise, refuse) yields the payoff pair (-1,-1).
2. The strategy pair (raise, accept) yields the payoff pair (7,3).
3. The strategy pair (maintain, refuse) yields the payoff pair (5,5).
4. The strategy pair (maintain, accept) yields the payoff pair (5,5).

It is convenient to form a **matrix** in which Russia's strategies correspond to the rows of that matrix and Ukraine's strategies correspond to the columns of that matrix.

		Ukraine	
		refuse	accept
Russia	raise		
	maintain		

GAME II-1

Notice that each cell in the matrix corresponds to a strategy pair:

1. The upper-left-hand cell corresponds to (raise, refuse).
2. The upper-right-hand cell corresponds to (raise, accept).
3. The lower-left-hand cell corresponds to (maintain, refuse).
4. The lower-right-hand cell corresponds to (maintain, accept).

Finally, recall that each strategy pair determines a payoff pair. Therefore, for each strategy pair, let's enter the corresponding payoff pair into the cell that matches the strategy pair.

For (raise, refuse) we must enter the payoff pair (-1,-1).

Russia \ Ukraine	refuse	accept
raise	(-1,-1)	
maintain		

GAME II-1

For (raise, accept) we enter the payoff pair (7,3).

		Ukraine	
		refuse	accept
Russia	raise	(-1,-1)	(7,3)
	maintain		

GAME II-1

For (maintain,refuse) we enter the payoff pair (5,5).

		Ukraine	
		refuse	accept
Russia	raise	(-1,-1)	(7,3)
	maintain	(5,5)	

Game II-1

For (maintain, accept) we enter the payoff pair (5,5).

		Ukraine	
		refuse	accept
Russia	raise	(-1,-1)	(7,3)
	maintain	(5,5)	(5,5)

Game II-1

The game represented in the **game matrix** Game II-1 is the same as the one given by the extensive form Game I-3.

Whenever we represent a game by listing the players, their strategies and the payoffs corresponding to each combination of strategies, we call this the **strategic form representation** of the game.

Any game has both an extensive-form representation and a strategic-form representation.

The strategic-form representation of a game is unique.

How does the backward-induction solution of the Russia-Ukraine game, (raise,accept), fare in the strategic form of that game?

		Ukraine	
		refuse	accept
Russia	raise	(-1,-1)	(7,3)
	maintain	(5,5)	(5,5)

Game II-1

We can ask two questions:

1. Could Russia gain by unilaterally deviating from the backward-induction solution?
2. Could Ukraine gain by unilaterally deviating from the backward induction solution?

If Russia unilaterally deviates from the backward-induction solution, it reduces its payoff from 7 to 5.

Russia \ Ukraine	refuse	accept
raise	(-1,-1)	(7,3)
maintain	(5,5)	(5,5)

GAME II-1

If Ukraine unilaterally deviates from the backward-induction solution, it reduces its payoff from 3 to -1.

		Ukraine	
		refuse	accept
Russia	raise	(-1,-1)	(7,3)
	maintain	(5,5)	(5,5)

GAME II-1

We conclude that the backward-induction solution of the Russia-Ukraine game is **robust against unilateral deviations** by either player.

An alternative way to express the same idea: The strategies **raise** for Russia and **accept** for Ukraine in Game II-1 are **mutual best replies**.

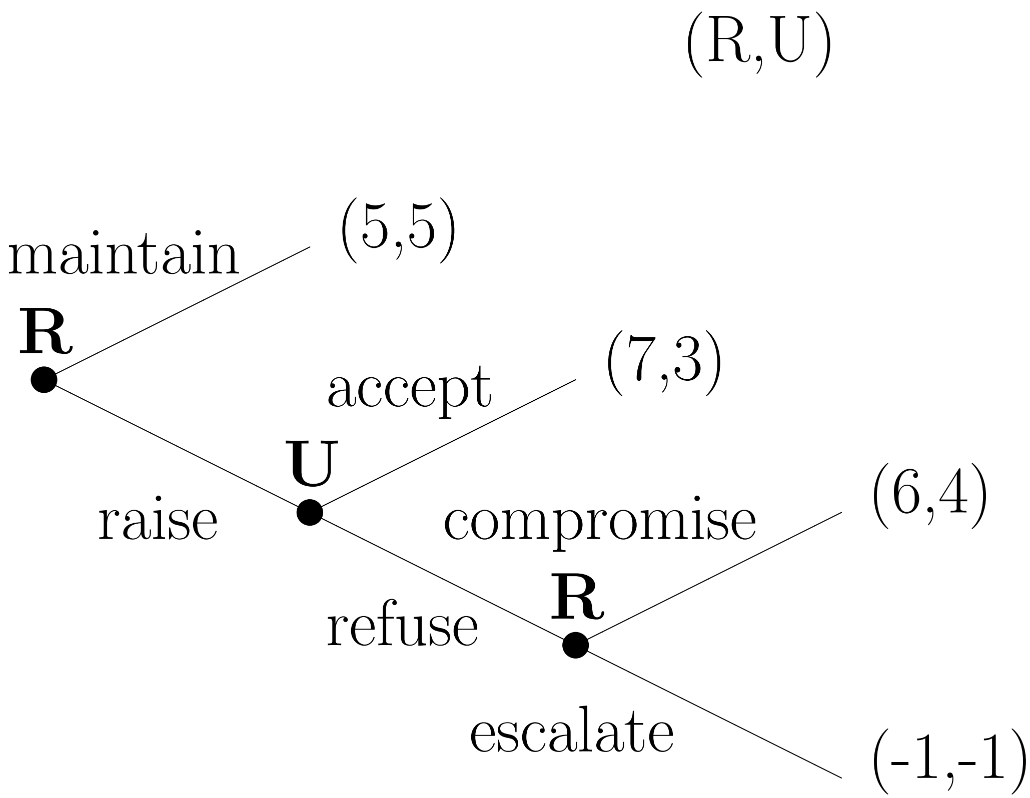
1. If Russia expects Ukraine to choose "accept", Russia has no better strategy than to choose "raise".
2. If Ukraine expects Russia to choose "raise", Ukraine has no better strategy than to choose "accept".

In a two-player game a pair of strategies that are **mutual best replies** is called a **Nash equilibrium**.

Note that every backward-induction solution is a Nash equilibrium.

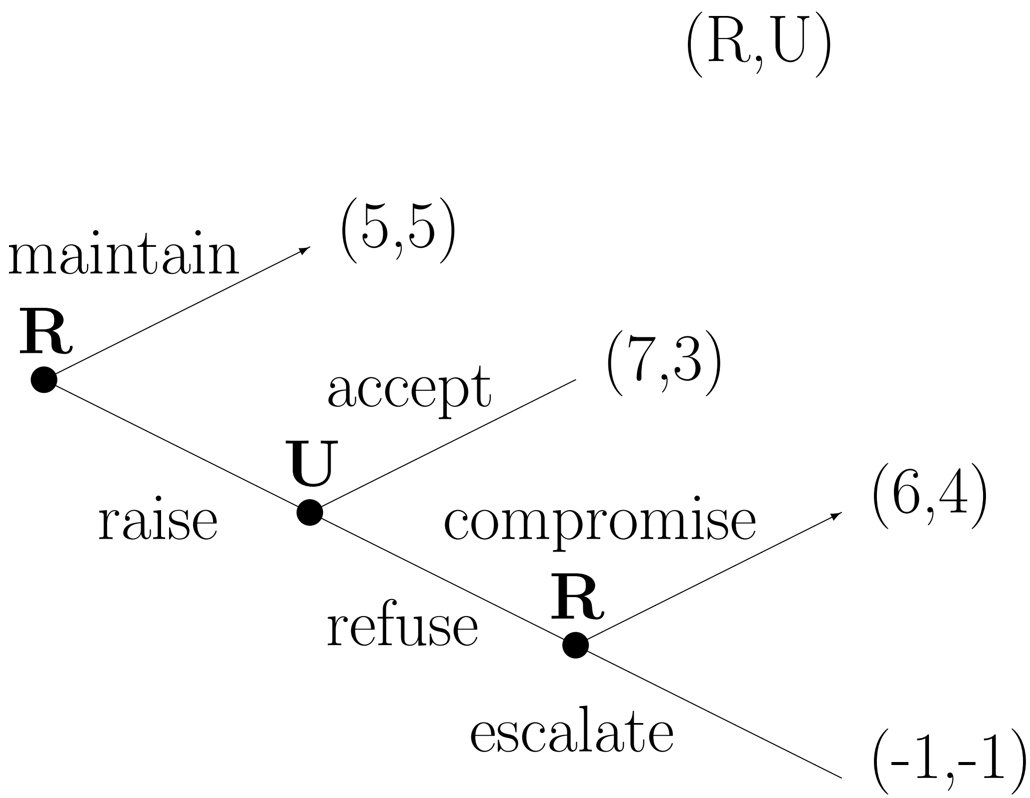
Some reasons for why **Nash equilibrium is attractive**:

1. If we observe social institutions that are stable over long periods of time, they must be supported by Nash equilibria. Otherwise, someone has an incentive to deviate, which would upset the institution.
2. If someone publicly proposed how to behave in a strategic situation, the proposal would have to be a Nash equilibrium. Otherwise there would be at least one individual with an incentive to ignore the proposal and to deviate from it.
3. If a theorist made a prediction for a strategic situation, the prediction would have to form a Nash equilibrium for everyone to believe it.



Game I-4

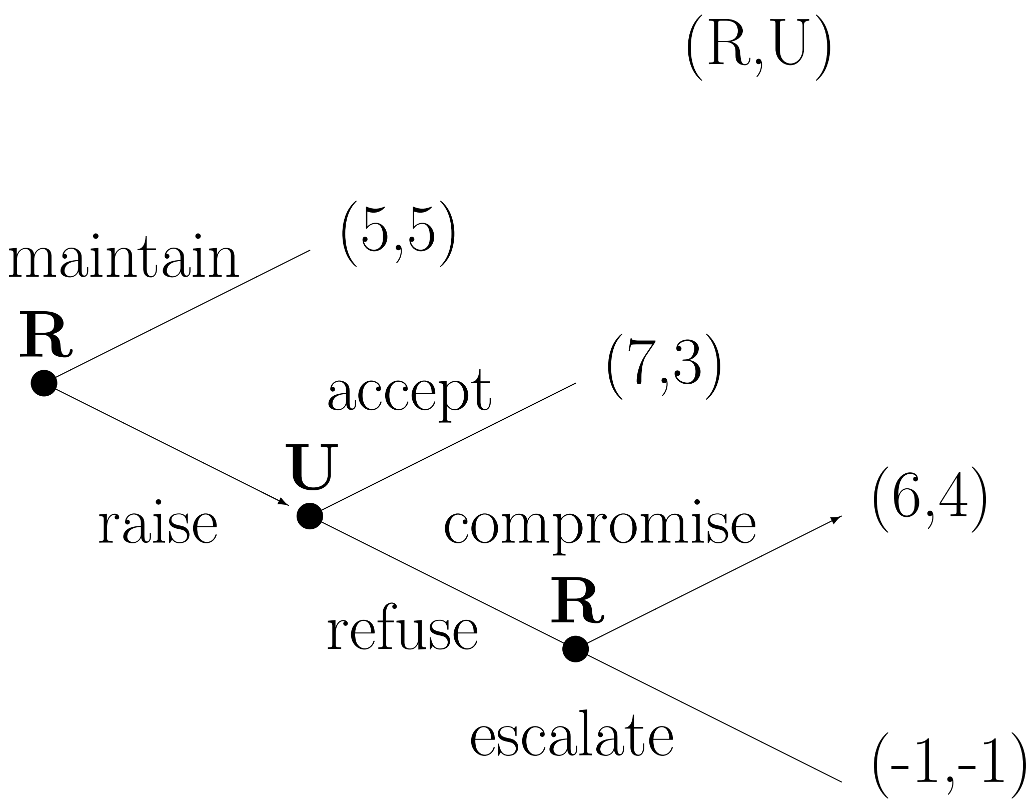
We saw that in the Game I-4, Ukraine has two strategies, **accept** and **refuse**, whereas Russia has four strategies: **(raise, compromise)**, **(raise, escalate)**, **(maintain, compromise)** and **(maintain, escalate)**.



GAME I-4

One way to indicate Russia's strategy **(maintain, compromise)** in the extensive form is to use arrows at each of Russia's decision nodes to mark the choices that the strategy **(maintain, compromise)** would induce at those nodes.

The strategy **(raise, compromise)** would be represented as follows:



GAME I-4

One easily checks that there are four different ways of assigning arrows to actions as all of Russia's decision nodes.

Each such assignment corresponds to a strategy.

Game I-4 in Strategic form:

Ukraine

Russia	refuse	accept
(raise, compromise)	(6,4)	(7,3)
(raise, escalate)	(-1,-1)	(7,3)
(maintain, compromise)	(5,5)	(5,5)
(maintain, escalate)	(5,5)	(5,5)

Game II-2

Again, we can identify the backward-induction solution of Game I-4 in its strategic-form representation Game II-2 and check that it is a Nash equilibrium:

		Ukraine: refuse	Ukraine: accept
Russia	(raise, compromise)	(6,4)	(7,3)
	(raise, escalate)	(-1,-1)	(7,3)
	(maintain, compromise)	(5,5)	(5,5)
	(maintain, escalate)	(5,5)	(5,5)

GAME II-2

Note that Russia is best responding because its payoff 6 in the backward-induction solution exceeds all payoffs that it can achieve by unilaterally deviating, -1 and 5.

Russia	Ukraine: refuse	Ukraine: accept
(raise, compromise)	(6,4)	(7,3)
(raise, escalate)	(-1,-1)	(7,3)
(maintain, compromise)	(5,5)	(5,5)
(maintain, escalate)	(5,5)	(5,5)

Game II-2

Ukraine is best responding because its payoff 4 in the backward-induction solution exceeds all payoffs that it can achieve by unilaterally deviating, i.e. 3.

Russia \ Ukraine	refuse	accept
(raise, compromise)	(6,4)	(7,3)
(raise, escalate)	(-1,-1)	(7,3)
(maintain, compromise)	(5,5)	(5,5)
(maintain, escalate)	(5,5)	(5,5)

GAME II-2

This confirms our general claim that a solution obtained by backward induction is always a Nash equilibrium.

A nice property of Nash equilibrium as a solution concept for games is that it applies also in situations where the backward-induction solution is not available.

A Trading Game

1. Jack and Jill each operate organic farms at opposite ends of a small town.
2. Jack specializes in vegetables, Jill in fruit.
3. Jack and Jill independently and simultaneously decide whether or not to sell their produce in the weekly farmers' market.
4. If Jack decides not to sell, he obtains a payoff of 2 that reflects the benefit of his time spent either on the farm or in leisure activities.

5. If Jack decides to sell, his payoff depends on whether or not Jill decides to sell as well:
   (a) If Jill does not show up at the market, Jack gets to sell some of his produce to other customers for a payoff 5.
   (b) If Jill does show up at the market, there is an added benefit from being able to sell vegetables to her and from buying fruit from her. In this case Jacks payoff from attending the market is 7.
6. Jill's situation similar. If she does not attend the market, her payoff is 2. If she does attend the market, she gets 5 if Jack does not show up and 7 if he does.

In the **trading game** there are two players, Jack and Jill, each of whom has two strategies, **sell** and **don't sell**.

Since the decisions are simultaneous, we cannot solve the trading game by backward induction.

The strategic-form representation of the trading game:

		Jill: sell	Jill: don't sell
Jack	sell	(7,7)	(5,2)
	don't sell	(2,5)	(2,2)

Game II-3

One easily checks that the pair of strategies **(sell,sell)** is the unique Nash equilibrium of the trading game.

		Jill: sell	Jill: don't sell
Jack	sell	(7,7)	(5,2)
	don't sell	(2,5)	(2,2)

GAME II-3

Notice that in the trading game, the unique Nash equilibrium also yields each player his/her maximal payoff in the game. This is an example of **Adam Smith's (1723 - 1790)** **invisible hand** in action: In a market environment, each agent pursuing only his own interest leads to a socially desirable outcome.

In the trading game, it is clear that the unique Nash equilibrium outcome is desirable because for every player it achieves that player's maximal possible payoff in the game.

This is often too much ask for: Suppose, we modify our trading game by making Jack and Jill competitors. Suppose, for example, that now both grow vegetables. As a consequence, each now prefers that the other does not attend the market.

We can capture these preference by reducing each player's payoff from the strategy pair **(sell,sell)** from 7 to 4.

		Jill	
		sell	don't sell
Jack	sell	(4,4)	(5,2)
	don't sell	(2,5)	(2,2)

Game II-4

Notice that in Game II-4, the strategy pair **(sell,sell)** remains the unique Nash equilibrium.

While the strategy pair **(sell,sell)** now fails to maximize every player's payoff, since $4 < 5$, it remains socially attractive in the following sense: **None of the other three strategy pairs makes both players better off than the strategy pair (sell,sell)**.

In a two-player game, we say that a strategy pair is **efficient** if it is impossible to find another strategy pair that improves at least one player's payoff, without making any player worse off.

In Game II-3, the unique Nash-equilibrium, (sell,sell), is also the unique efficient strategy profile (btw, we refer to any complete list of strategies, one for each player, as a strategy profile).

In Game II-4, the unique Nash-equilibrium, (sell,sell), is also efficient. In this case, however, there are other efficient profiles. The two profiles (sell, don't sell) and (don't sell, sell) are efficient as well.

Most importantly: **In both Game II-3 and Game II-4, Nash equilibrium implies efficiency.**

Summary: There is a class of games in which Nash equilibrium implies efficiency.

Caveats and remarks:

1. **Not every efficient strategy profile is a Nash equilibrium**; e.g., in Game II-4 the profile (sell, don't sell) is efficient but not a Nash equilibrium
2. **Not every Nash equilibrium is efficient**; e.g., in the fishing game of Homework 1, the Nash equilibrium that is obtained by backward induction is not efficient.

Bellum omnium contra omnes

Before Adam Smith, Thomas Hobbes, April 5, 1588-December 4, 1679) described life in the state of nature as a "war of all against all."

Here the state of nature describes society before the institution of some authority.

In the Hobbesian state of of nature, there are no well-defined property rights and there is no protection by the law.

As a result, unlike in the voluntary market exchanges of Adam Smith, which benefit all participants, interactions in the state of nature frequently involve the pursuit of individual self-interest at the expense of others.

In the Hobbesian state of nature, the war of all against all is not in everyone's best interest.

The simplest formal representation of the dilemma faced by society in the Hobbesian state of nature is the famous **Prisoners' Dilemma** game.

The Hobbesian State of Nature as a Prisoners' Dilemma

1. Simplify a society to consist of only two individuals, Attila the Hun and Ghengis Khan
2. Each member of this society has two strategies, "prepare for war" and "pursue peace".

3. If both Attila and Ghengis opt for peace, they get to enjoy a comfortable life, with a payoff of 5 for each of them.
4. If Attila the Hun chooses war, and Ghengis Khan chooses peace, then Attila captures Ghengis's resources, which results in a payoff of 8 for Attila and 0 for Ghengis.
5. If Ghengis Khan chooses war, and Attila the Hun chooses peace, then Ghengis captures Attila's resources, which results in a payoff of 8 for Ghengis and 0 for Attila.
6. If both prepare for war, both will spend some of their resources in preparing for war, but neither will risk losing all of their resources to their opponent. Let the common payoff in this case be 1.

The Prisoners' Dilemma

Attila \ Ghengis	peace	war
peace	(5,5)	(0,8)
war	(8,0)	(1,1)

Game II-5

The Prisoners' Dilemma has a unique Nash equilibrium, viz. (war,war)

		Ghengis: peace	Ghengis: war
Attila	peace	(5,5)	(0,8)
	war	(8,0)	(1,1)

GAME II-5

Comments on the Prisoner's Dilemma:

1. The Prisoners' Dilemma is of interest because there is such an **obvious tension between the game theoretic prediction and social desirability**.
2. **In the Prisoners' Dilemma, the game theoretic solution is unassailable. Attempts to "solve" the Prisoners' Dilemma**, when they are not changing the game, **are simply silly**.
3. There is a plethora of social situations that resemble the Prisoners' Dilemma:
   (a) global warming
   (b) having the UN adopt a common policy toward Iran
   (c) doing well on an exam when the exam is strictly graded on a curve

No matter, how unpalatable the predicted outcome of the Prisoners' Dilemma is, given the payoffs, the strategy choice "war" is unambiguously the right one.

The Prisoners' Dilemma is special in that for each player the optimal strategy choice is independent of the expected behavior of the other player.

Attila's payoffs in the Prisoners' Dilemma

		Ghengis peace	Ghengis war
Attila	peace	5	0
Attila	war	8	1

Game II-5

Note that in each column the payoff in the bottom row exceeds the payoff from the top row.

Attila's strategy "war" **strictly dominates** the strategy "peace" because it yields a strictly higher payoff than "peace" regardless of Ghengis's strategy.

In the following game, the situation is similar

Attila \ Ghengis	peace	war
peace	(5,5)	(0,8)
war	(8,0)	(0,0)

GAME II-6

In Game II-6 Attila's strategy "war" **(weakly) dominates** the strategy "peace" because regardless of Ghengis's strategy choice, "war" never yields a lower payoff than "peace," and sometimes "war" leads to a strictly higher payoff than "peace."

The Prisoners' Dilemma once more

1. The solution in the Prisoners' Dilemma can be obtained by **eliminating strictly dominated strategies.**
2. Each player in the Prisoners' Dilemma can determine his optimal choice entirely without reasoning about the other player.
3. **There is not a shadow of a doubt about behavior in the Prisoners' Dilemma.**
4. This is precisely what makes the Prisoners' Dilemma so compelling: There is an **inescapable tension** between individual incentives and socially desirable outcomes.

A Three-Player Game

One player chooses rows, **up** and **down**; one player chooses columns, **left** and **right**; one player chooses matrices, **east** and **west**. Each payoff profile adheres to the ordering (row, column, matrix).

	east: left	east: right	west: left	west: right
up	(5,5,5)	(0,8,0)	(0,0,8)	(0,1,1)
down	(8,0,0)	(1,1,0)	(1,0,1)	(1,1,1)

Game II-7

Once more, there is a unique strategy profile that remains after deleting strictly dominated strategies.

	east		west	
	left	right	left	right
up	(5,5,5)	(0,8,0)	(0,0,8)	(0,1,1)
down	(8,0,0)	(1,1,0)	(1,0,1)	(1,1,1)

GAME II-7

The strategy profile (down, right, west) that we found by eliminating strictly dominated strategies is also the only Nash equilibrium profile.

General principle: Whenever a unique strategy profile survives elimination of strictly dominated strategies, this strategy profile is a Nash equilibrium.