Game Theory **Andreas Blume**
Principles **8th Homework - Solutions**

1. Consider the following game

	A	B
A	(0,0)	(5,8)
B	(8,5)	(0,0)

(a) Find all Nash equilibria.

i. There are two pure-strategy equilibria, (A, B) and (B, A), and one mixed-strategy equilibrium. The mixed-strategy equilibrium is symmetric: Each player uses strategy A with probability $p = \frac{5}{13}$.

(b) Is there an ESS?

i. Since an ESS has to be symmetric, only the mixed-strategy equilibrium, with $p = \frac{5}{13}$, is a candidate for an ESS.

ii. Denote any mutant mixed strategy by p'. Using the notation introduced in class, we have

$$E(p, p) = E(p', p).$$

Hence, in order to have an ESS, we need

$$E(p, p') > E(p', p').$$

We have

$$\begin{aligned} E(p, p') &= p(1 - p')5 + (1 - p)p'8 \\ E(p', p') &= p'(1 - p')5 + (1 - p')p'8 \end{aligned}$$

Therefore,

$$E(p, p') - E(p', p') = (p - p')(1 - p')5 + (p' - p)p'8$$

and we have an ESS provided the sign of this expression is positive for any $p' \neq p$. To check the sign observe that

$$p' > p \Rightarrow p'8 > (1 - p')5,$$

and

$$p' < p \Rightarrow p'8 < (1-p')5.$$

Hence, in either case, the sign of

$$E(p,p') - E(p',p')$$

is positive. Therefore, the mixed strategy equilibrium is indeed an ESS.

(c) Suppose the game is played following the observation of either a red or a green light lighting up, with probability $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Find the extensive form representation of this game. Find the strategic form representation. Find all pure-strategy Nash equilibria.

The strategic form is given below with pure-strategy Nash equilibria indicated in blue.

	(A,A)	(A,B)	(B,A)	(B,B)
(A,A)	$0,0$	$\frac{10}{3},\frac{16}{3}$	$\frac{8}{3},\frac{5}{3}$	$\mathbf{5,8}$
(A,B)	$\frac{16}{3},\frac{10}{3}$	$0,0$	$\mathbf{7,6}$	$\frac{5}{3},\frac{8}{3}$
(B,A)	$\frac{5}{3},\frac{8}{3}$	$\mathbf{6,7}$	$0,0$	$\frac{10}{3},\frac{16}{3}$
(B,B)	$\mathbf{8,5}$	$\frac{8}{3},\frac{5}{3}$	$\frac{16}{3},\frac{10}{3}$	$0,0$

(d) Suppose the game is played following simultaneous announcements by both players, where players can announce either H or T. What is the highest expected payoff players can achieve in a symmetric equilibrium?

The simultaneous pre-play announcement allows players to correlate their actions. This is similar to the effect of the red and green lights in the previous question. There is an equilibrium in which both players randomize over H and T, choosing each with equal probability, in the communication phase. Following the communication phase, they play (A,B) if their communication choices match, and (B,A) otherwise. In this equilibrium both players have an expected payoff of $\frac{13}{2}$. Evidently, this is the highest symmetric payoff that can be achieved in the game and therefore the highest symmetric equilibrium payoff.

(e) What do the last two questions tell us about changing the game and what inference might one draw regarding quantum games?

Allowing for correlation, signal dependence, communication etc frequently enlarges both the sets of equilibria and the sets of equilibrium payoffs. This is similar to the effect of giving players access to quantum strategies. The move to quantum strategies is more dramatic because it permits a player to correlate his actions with those of the other player, even is the other player is restricted to conventional strategies.