Game Theory **Andreas Blume**
Principles **5th Homework**

1. Suppose that you are a physician and have just received the results of a test for disease X in one of your patients. You know that in the general population the frequency of disease X is 15 in 1 million. The test is 95% accurate, i.e. if you have the disease, the test will be positive with probability .95 and if you don't have the disease the test will be negative with probability .95. In the case of your patient the test is positive. How likely is it that your patient has disease X? (Hint: Use Bayes's rule to calculate the probability of having disease X conditional on the test being positive.)

2. Consider the policy advice game with the following payoff matrix. Suppose that the types s and t of the policy maker are equally likely and that the expert can send either message m or n. Recall that each cell of this payoff matrix records both the expert's and the policy maker's payoff from the corresponding type-action combination.

	a	b	c
s	$3, 3$	$0, 0$	$2, 2$
t	$0, 0$	$3, 3$	$2, 2$

   (a) If type s sends message m with probability .6 and type t sends message n with probability .55, what is the posterior belief of the policy maker when he observes message m? Use Bayes's rule to find your answer.

   (b) Is there a Nash equilibrium in which type s sends message m with probability .6 and type t sends message n with probability .55?

   (c) Is there a Nash equilibrium in which type s sends message m with probability .6 and type t sends message n with probability .9?

   (d) What's going on?

3. Consider the policy advice game with the following payoff matrix. Suppose that the types s and t of the policy maker are equally likely and that the expert can send either message m or n. In this game the policy maker has the choice among five actions, a, b, c, d and e.

	a	b	c	d	e
s	$1, 5$	$0, -5$	$5, 4$	$5, 0$	$2, 3$
t	$0, -5$	$1, 5$	$5, 0$	$5, 4$	$2, 3$

   (a) Show that this game has a *pooling equilibrium* in which both types of the expert send the same message.

   (b) Show that this game has a *separating equilibrium* in which the two types of the expert send distinct messages.

   (c) Which of the two equilibria above does the sender prefer? What's going on?

   (d) Show that there is an equilibrium in which type s sends message m with probability $\frac{3}{4}$ and type t sends message n with probability $\frac{3}{4}$.

4. Argue that in a policy advice game, the policy maker will always (regardless of the payoff matrix) prefer a separating equilibrium if there is one.