XI.—The Relation between Mathematics and Physics, By
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The physicist, in his study of natural phenomena, has two methods of
making progress: (1) the method of experiment and observation, and
(2) the method of mathematical reasoning. The former is just the collec-
tion of selected data; the latter enables one to infer results about experi-
ments that have not been performed. There is no logical reason why
the second method should be possible at all, but one has found in practice
that it does work and meets with remarkable success. This must be
ascribed to some mathematical quality in Nature, a quality which the
casual observer of Nature would not suspect, but which nevertheless
plays an important rôle in Nature's scheme.

One might describe the mathematical quality in Nature by saying that
the universe is so constituted that mathematics is a useful tool in its
description. However, recent advances in physical science show that
this statement of the case is too trivial. The connection between mathe-
matics and the description of the universe goes far deeper than this, and
one can get an appreciation of it only from a thorough examination of
the various factors that make it up. The main aim of my talk to you
will be to give you such an appreciation. I propose to deal with how the
physicist's views on this subject have been gradually modified by the
succession of recent developments in physics, and then I would like to
make a little speculation about the future.

Let us take as our starting-point that scheme of physical science which
was generally accepted in the last century—the mechanistic scheme.
This considers the whole universe to be a dynamical system (of course
an extremely complicated dynamical system), subject to laws of motion
which are essentially of the Newtonian type. The rôle of mathematics
in this scheme is to represent the laws of motion by equations, and to
obtain solutions of the equations referring to observed conditions.

The dominating idea in this application of mathematics to physics is
that the equations representing the laws of motion should be of a simple
form. The whole success of the scheme is due to the fact that equations
of simple form do seem to work. The physicist is thus provided with a
principle of simplicity, which he can use as an instrument of research. If he obtains, from some rough experiments, data which fit in roughly with certain simple equations, he infers that if he performed the experiments more accurately he would obtain data fitting in more accurately with the equations. The method is much restricted, however, since the principle of simplicity applies only to fundamental laws of motion, not to natural phenomena in general. For example, rough experiments about the relation between the pressure and volume of a gas at a fixed temperature give results fitting in with a law of inverse proportionality, but it would be wrong to infer that more accurate experiments would confirm this law with greater accuracy, as one is here dealing with a phenomenon which is not connected in any very direct way with the fundamental laws of motion.

The discovery of the theory of relativity made it necessary to modify the principle of simplicity. Presumably one of the fundamental laws of motion is the law of gravitation which, according to Newton, is represented by a very simple equation, but, according to Einstein, needs the development of an elaborate technique before its equation can even be written down. It is true that, from the standpoint of higher mathematics, one can give reasons in favour of the view that Einstein's law of gravitation is actually simpler than Newton's, but this involves assigning a rather subtle meaning to simplicity, which largely spoils the practical value of the principle of simplicity as an instrument of research into the foundations of physics.

What makes the theory of relativity so acceptable to physicists in spite of its going against the principle of simplicity is its great mathematical beauty. This is a quality which cannot be defined, any more than beauty in art can be defined, but which people who study mathematics usually have no difficulty in appreciating. The theory of relativity introduced mathematical beauty to an unprecedented extent into the description of Nature. The restricted theory changed our ideas of space and time in a way that may be summarised by stating that the group of transformations to which the space-time continuum is subject must be changed from the Galilean group to the Lorentz group. The latter group is a much more beautiful thing than the former—in fact, the former would be called mathematically a degenerate special case of the latter. The general theory of relativity involves another step of a rather similar character, although the increase in beauty this time is usually considered to be not quite so great as with the restricted theory, which results in the general theory being not quite so firmly believed in as the restricted theory.

We now see that we have to change the principle of simplicity into a
The relation between mathematics and physics. The position very transversal, philosophically, as it goes beyond the description of the universe. It is not easily distinguished from the remainder. However, it is clear that the mathematician, in his work, is concerned with the internal consistency and completeness of the system he is constructing. The mathematician, on the other hand, is concerned with the logical structure of the system and the way it relates to the real world. The two are connected, but they are not the same.

According to the author, the mathematician's work is not just about creating beautiful structures, but also about understanding the nature of reality. The mathematician's work is a way of understanding the world, and it is this understanding that is of greatest value. The mathematician's work is not just about creating beautiful structures, but also about understanding the nature of reality. The mathematician's work is a way of understanding the world, and it is this understanding that is of greatest value.

The author also discusses the role of mathematics in the sciences. He argues that mathematics is the language of science, and that it is through mathematics that we can understand the world around us. The author concludes by emphasizing the importance of mathematics in our understanding of the universe, and the role that it plays in our ability to understand the world around us.
must be included, together with the initial conditions, in that part of the
description of the universe outside mathematical theory.

The increase thus arising in the non-mathematical part of the descrip-
tion of the universe provides a philosophical objection to quantum
mechanics, and is, I believe, the underlying reason why some physicists
still find it difficult to accept this mechanics. Quantum mechanics
should not be abandoned, however, firstly, because of its very widespread
and detailed agreement with experiment, and secondly, because the
indeterminacy it introduces into the results of observations is of a kind
which is philosophically satisfying, being readily ascribable to an inescap-
able crudeness in the means of observation available for small-scale
experiments. The objection does show, all the same, that the foundations
of physics are still far from their final form.

We come now to the third great development of physical science of the
present century—the new cosmology. This will probably turn out to be
philosophically even more revolutionary than relativity or the quantum
theory, although at present one can hardly realize its full implications.
The starting-point is the observed red-shift in the spectra of distant
heavily bodies, indicating that they are receding from us with velocities
proportional to their distances.* The velocities of the more distant ones
are so enormous that it is evident we have here a fact of the utmost im-
portance, not a temporary or local condition, but something fundamental
for our picture of the universe.

If we go backwards into the past we come to a time, about $2 \times 10^7$
years ago, when all the matter in the universe was concentrated in a very
small volume. It seems as though something like an explosion then
took place, the fragments of which we now observe still scattering out-
wards. This picture has been elaborated by Lemaitre, who considers
the universe to have started as a single very heavy atom, which under-
went violent radioactive disintegrations and so broke up into the
present collection of astronomical bodies, at the same time giving off
the cosmic rays.

With this kind of cosmological picture one is led to suppose that
there was a beginning of time, and that it is meaningless to inquire into
what happened before then. One can get a rough idea of the geometrical
relationships this implies by imagining the present to be the surface of a

* The recession velocities are not strictly proved, since one may postulate some
other cause for the spectral red-shift. However, the new cause would presumably be
equally drastic in its effect on cosmological theory and would still need the introduction
of a parameter of the order $2 \times 10^7$ years for its mathematical discussion, so it would
probably not disrupt the essential ideas of the argument in the text.
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Mathematical theory applies to a description of the physical universe. The part to which it does not apply has suffered an increase with the arrival of quantum mechanics and a decrease with the arrival of the new cosmology, but has always remained.

This feature is so unsatisfactory that I think it safe to predict it will disappear in the future, in spite of the startling changes in our ordinary ideas to which we should then be led. It would mean the existence of a scheme in which the whole of the description of the universe has its mathematical counterpart, and we must suppose that a person with a complete knowledge of mathematics could deduce, not only astronomical data, but also all the historical events that take place in the world, even the most trivial ones. Of course, it must be beyond human power actually to make these deductions, since life as we know it would be impossible if one could calculate future events, but the methods of making them would have to be well defined. The scheme could not be subject to the principle of simplicity since it would have to be extremely complicated, but it may well be subject to the principle of mathematical beauty.

I would like to put forward a suggestion as to how such a scheme might be realized. If we express the present epoch, $2 \times 10^9$ years, in terms of a unit of time defined by the atomic constants, we get a number of the order $10^{40}$, which characterizes the present in an absolute sense. Might it not be that all present events correspond to properties of this large number, and, more generally, that the whole history of the universe corresponds to properties of the whole sequence of natural numbers? At first sight it would seem that the universe is far too complex for such a correspondence to be possible. But I think this objection cannot be maintained, since a number of the order $10^{40}$ is excessively complicated, just because it is so enormous. We have a brief way of writing it down, but this should not blind us to the fact that it must have excessively complicated properties.

There is thus a possibility that the ancient dream of philosophers to connect all Nature with the properties of whole numbers will some day be realized. To do so physics will have to develop a long way to establish the details of how the correspondence is to be made. One hint for this development seems pretty obvious, namely, the study of whole numbers in modern mathematics is inextricably bound up with the theory of functions of a complex variable, which theory we have already seen has a good chance of forming the basis of the physics of the future. The working out of this idea would lead to a connection between atomic theory and cosmology.

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